Random Matrices, Black holes, and the Sachdev-Ye-Kitaev model

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Verbaarschot
Stony Brook

Bermúdez
Leiden

Tezuka
Kyoto

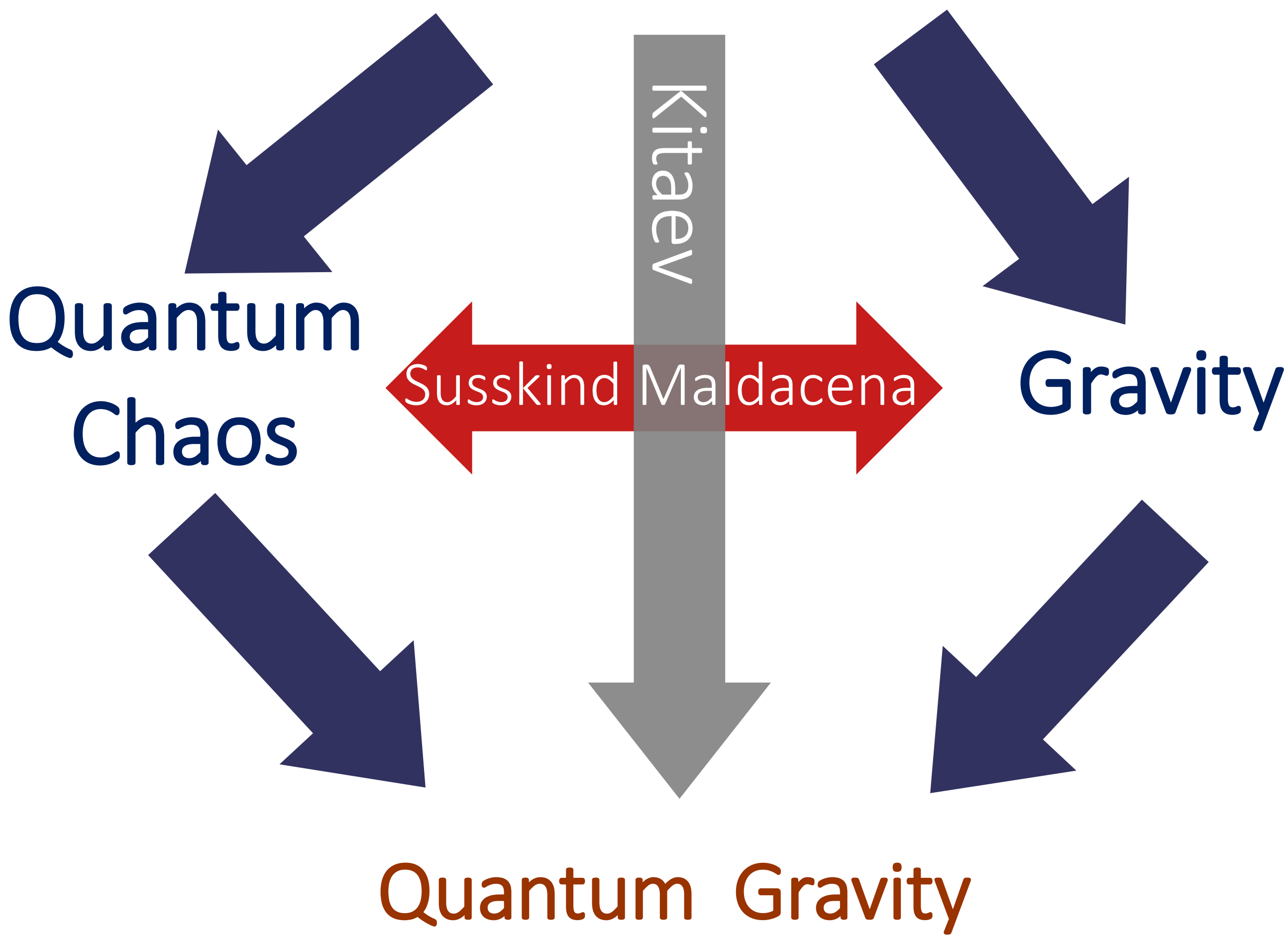
Loureiro
Cambridge

Yiyang Jia
Stony Brook

Phys. Rev. D 97, 106003 (2018)
Phys. Rev. D 94, 126010 (2016)
Phys. Rev. D 96, 066012 (2017)
arXiv:1801.03204
JHEP 04(2018)146
PRL 120, 241603 (2018)

SYK model

0D, N Majoranas, Infinite Range Interactions



Verbaarschot, AGG: RMT is a generic property of quantum gravity $t>t_H$

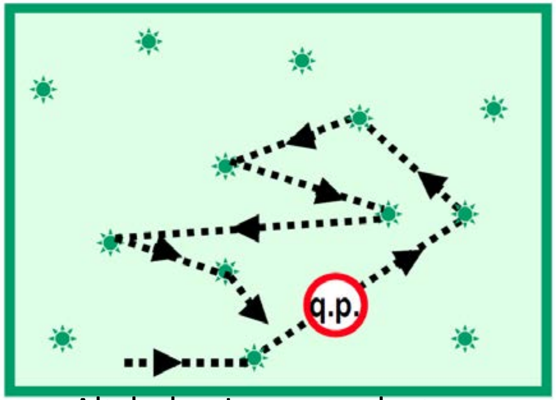
What quantum chaos has to do with (quantum) gravity?

Quantum chaos?

Role of classical chaos in quantum mechanics

Disordered system

Larkin, Ovchinnikov,
Soviet Physics JETP 28, 1200 (1969)



Altshuler, Lancaster lectures

$\langle p_z(t) p_z(0) \rangle \propto e^{-t/\tau}$

τ Relaxation time

Kicked rotors: Zaslavsky, Berman, Physica 91A 450 (1978)

$$\langle [p_z(t), p_z(0)]^2 \rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2 \rangle \propto \hbar^2 \exp(\lambda t)$$

$\tau \ll t < t_E \sim \log \hbar^{-1}/\lambda$ Chaotic

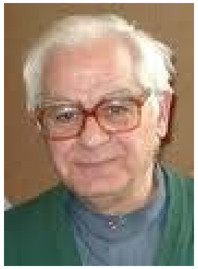
$t_E \propto \hbar^{\alpha}$ $\alpha < 0$ Integrable

Chaos and random matrix theory

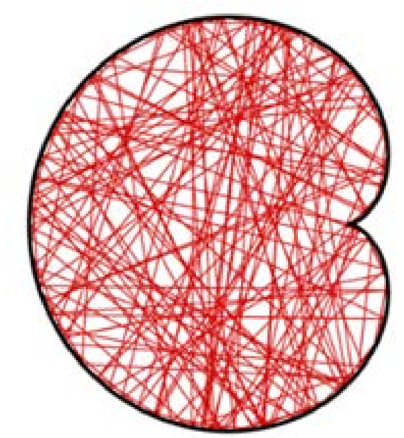
Bohigas-Giannoni-Schmit conjecture

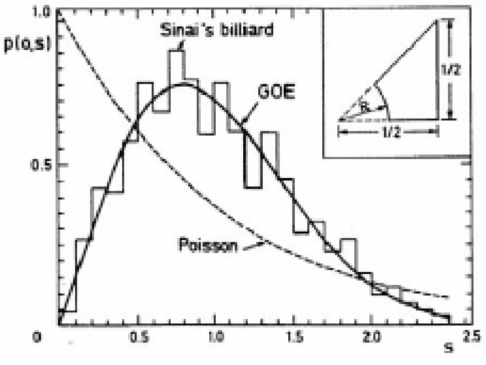
PRL 52, 1 (1984)

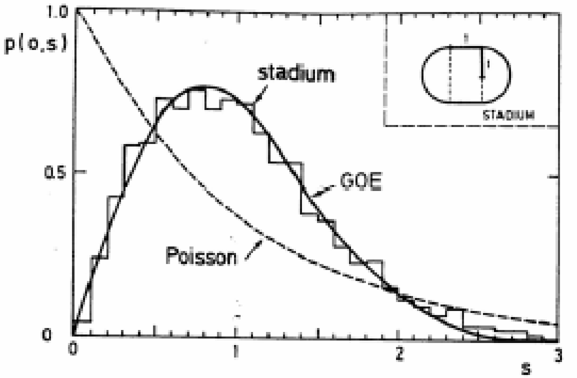
Proof PRO Sieber, Richter, 2001



Oriol Bohigas







Quantum Chaos ⟺ RMT correlations

Universality $t > t_H$ Heisenbserg time

$$\hat{G}_{\mathrm{R,A}} = (E - \hat{H} \pm i\eta)^{-1}$$

$$(E - \hat{H} + i\eta)^{-1}_{kl} = -i\frac{\int [\mathrm{d}\phi^*\mathrm{d}\phi]\phi_k\phi_l^* \exp\{i\sum_{ij}\phi_i^*[(E+i\eta)\delta_{ij} - H_{ij}]\phi_j\}}{\int [\mathrm{d}\phi^*\mathrm{d}\phi] \exp\{i\sum_{ij}\phi_i^*[(E+i\eta)\delta_{ij} - H_{ij}]\phi_j\}}$$

1982-84: Grassmannian variables can help

Efetov

$$\chi_k\chi_l = -\chi_l\chi_k$$

$$I = \int \exp\left(-\chi^+ A\chi\right)\prod_{i=1}^{n} d\chi_i^* d\chi_i = Det A$$

$$(E - \hat{H})^{-1}_{kl} = -i\int [\mathrm{d}\Phi^*\mathrm{d}\Phi] S_k S_l^* \exp\{i\sum_{ij}\Phi_i^\dagger[E\delta_{ij} - H_{ij}]\Phi_j\}$$

$$\Phi^\dagger = (S_1^*, \ldots, S_n^*, \chi_1^*, \ldots, \chi_n^*)$$

$$\left\langle \exp(i\sum_{ij}\Phi_i^\dagger H_{ij}\Phi_j)\right\rangle = \exp\left\{-\frac{1}{2N}\sum_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i)\right\}$$

Disorder Is integrated!!

Disordered metals in d > 2 ⟺ RMT correlations

Chaos in black-hole physics

Black Holes are fastest scramblers in nature

Sekino, Susskind,JHEP 0810:065,2008

P. Hayden, J. Preskill, JHEP 0709 (2007) 120

(Quantum) black hole physics	AdS/CFT	Strongly coupled (quantum) QFT

Field theory dual also fastest scramblers

$$t_S \sim \log(N)$$

$t_S \sim \mathrm{t}_E$?

Hyperbolic billiards are quantum chaotic

Membrane paradigm

Drop a charged particle onto the horizon and compute the time for the charge to equilibrate

Rindler

$$ds^2 = -\rho^2 d\omega^2 + d\rho^2 + dx_\perp^2$$

$$\begin{aligned} \rho^2 &= z^2 - t^2 \\ z &= \rho \cosh \omega \\ t &= \rho \sinh \omega \end{aligned}$$

$$\omega \gg 1$$

$$\sigma = \frac{1}{4\pi\rho} E_\rho|_{\rho SH} = \frac{e}{4\pi \ell_p} \frac{\ell_p e^\omega}{[(\ell_p e^\omega)^2 + x_\perp^2]^{\frac{3}{2}}}$$

Spread of charge density $\Delta x \sim l_p e^\omega$

Scrambling time $\omega_S \sim \log R_s / l_p$ $\quad \omega = 2\pi t/\beta$

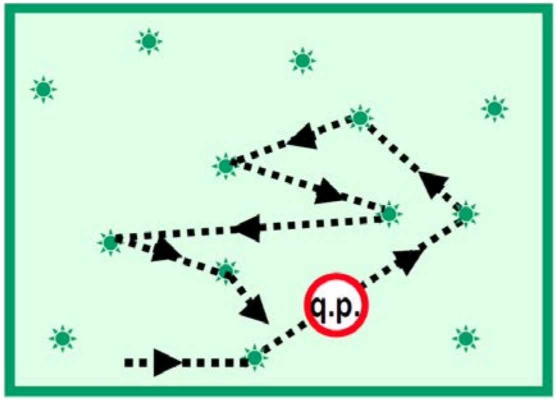
Far from horizon

$$t_S \sim \beta S^{2/d}$$

$$t_S \sim \beta \log S \sim \beta \log N$$

Quantum chaos in gravity/holography?

Maldacena, Shenker, Stanford arXiv:1503.01409



$$\tau \ll t < t_E \sim \log \hbar^{-1}/\lambda$$

Quantum Chaos

$$\langle [p_z(t), p_z(0)]^2 \rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2 \rangle \propto \hbar^2 \exp(\lambda t)$$

Field theory?

Black holes?

λ?

A bound on chaos arXiv:1503.01409

Maldacena, Shenker, Stanford

$$y^4 = \frac{1}{Z} e^{-\beta H} \qquad F(t) = \mathrm{tr}[yVyW(t)yVyW(t)]$$

$$t_* = \frac{\beta}{2\pi} \log N^2 \qquad F_d \equiv \mathrm{tr}[y^2 V y^2 V]\mathrm{tr}[y^2 W(t) y^2 W(t)]$$

$$t_d \ll t < t_* \qquad F_d - F(t) = \epsilon \exp \lambda_L t + \cdots \qquad \epsilon \sim 1/N^2$$

$$F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$$

$$\lambda \leq 2\pi T/\hbar$$

Black holes and its field theory dual saturate the bound

Causality constraints
+
Uncertainty relations

Berenstein, AGG
arXiv:1510.08870

$$p \leq e^{t/4MG}$$

How is this related to quantum information?

$$S \sim t/\tau$$

How universal?

$$\tau \geq \hbar/2\pi k_B T$$

QM induces entanglement but also limits its growth

SYK model and relation with (quantum) gravity

Before SYK: k-random body ensembles

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\le l, p\le q} \langle pq| V |kl\rangle a_p^\dagger a_q^\dagger a_l a_k$$

N fermions
m levels
2-body

Bohigas, Flores, French, Wong, 70's

$$m \gg N \quad \langle H^p \rangle \quad \rightarrow \quad \rho(E) \propto e^{-E^2/\sigma^2}$$

French, Mon, Annals of Physics 95, 90 (1975).

Two-level correlation function ⟷ Random Matrix

Verbaarschot, Zirnbauer 85

Weidenmuller, Izrailev, Benet

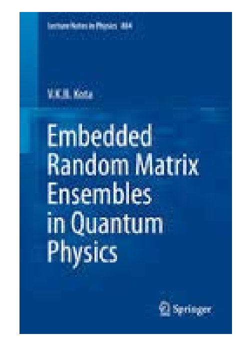
Quantum Chaos 70's-90's: QCD, Nuclear Physics and others

Level statistics

Metal-insulator transitions

Thermalization

Thouless time and other time scales



Kota

Quantum spin glasses

Heisenberg Spin-Chain

$$H = \frac{1}{\sqrt{\mathcal{N}N}} \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Infinite Range

Replica Trick

Large N

Many-body quantum chaos is not new!!!

Stability of magnetic order

Cuprates, spin-liquids ....

Quantum criticality

Sachdev, Ye, PRL. 70, 3339 (1993)

A. Georges, O. Parcollet, S. Sachdev
PRB 63, 134406 (2001)

Georges, Parcollet, PRB 58, 3794 (1998)

Finite zero T entropy

Holography dual?

S. Sachdev PRL 83, 74408 (2010)

Kitaev: "A simple model of quantum holography"

http://online.kitp.ucsb.edu/online/entangled15/kitaev/

No kinetic term $H = J_{ijkl}\psi_i\psi_j\psi_k\psi_l$

Majoranas $\{\psi_i, \psi_j\} = \delta_{ij}$

Gaussian $\langle J_{ijkl}^2 \rangle \sim \mathrm{J}^2/\mathrm{N}^3$

Strong coupling $\beta J \gg 1 \quad \tau J \gg 1$

A solvable finite model of quantum gravity

SYK = Sachdev-Ye-**Kitaev**

Correlation functions

Disorder average by replica trick

$$\langle Z(\beta)\rangle_J = \int DGD\Sigma\, e^{-N\, I(G,\Sigma)} \qquad G(\tau_1,\tau_2)\sim\langle\psi(\tau_1)\psi(\tau_2)\rangle$$

$$I(G,\Sigma) = -\frac{1}{2}\log\det(\partial_\tau - \Sigma) + \frac{1}{2}\int_0^\beta d\tau_1 d\tau_2\left[\Sigma(\tau_1,\tau_2)G(\tau_1,\tau_2) - \frac{J^2}{q}G(\tau_1,\tau_2)^q\right]$$

q = q/2-body interaction

$$N \to \infty \qquad \Sigma_* = J^2 G_*^{q-1} \qquad G_* = \frac{1}{\partial_\tau - \Sigma_*}$$

Self-consistent Schwinger-Dyson equations

Zero Temperature

$$J\tau, J\beta \gg 1 \quad \partial_\tau \to 0$$

Conformal (reparametrization invariant) in the IR limit

$$\Delta = 1/q$$

Finite Zero Temperature entropy

$$\frac{S_0}{N} = \frac{1}{2}\log 2 - \int_0^\Delta dx \pi \left(\frac{1}{2} - x\right) \tan \pi x$$

Georges, Parcollet 90's   Kitaev 2015

Also in some $NAdS_2$ background

Low temperature: Correction to conformal

Soft breaking of reparametrization invariance → Schwarzian action Goldstone modes

$$S = -N\frac{\alpha_S}{\mathcal{J}}\int d\tau \{f,\tau\} \qquad \{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

Residual SL(2,R)

Same as in $NAdS_2$

$$-\beta F \supset \frac{N\alpha_S}{\mathcal{J}}\int_0^\beta d\tau \left\{\tan\frac{\pi\tau}{\beta}, \tau\right\} = 2\pi^2\alpha_S\frac{N}{\beta\mathcal{J}}$$

Linear specific heat

J. Maldacena, D. Stanford, Phys. Rev. D 94, 106002 (2016)

1/N Quantum corrections

$$\frac{S}{N} = \frac{J^2(q-1)}{4} g \cdot (\tilde{K}^{-1} - 1) g$$

$$-\beta F \supset -\sum_{n=2}^{\infty} \log \frac{n}{\beta J} + \text{const} \to \#\beta J - \frac{3}{2} \log \beta J + \text{const}$$

$$\frac{\langle \psi_i(0)\psi_j(\tau)\psi_i(0)\psi_j(\tau)\rangle}{\langle \psi_i(0)\psi_i(0)\rangle\langle \psi_j(\tau)\psi_j(\tau)\rangle} \propto 1 + i\frac{\beta J}{N} e^{\frac{2\pi\tau}{\beta}}$$

SYK is quantum chaotic

SYK saturates Maldacena bound

SYK has a gravity dual

Gravity dual: Quantum N(ear) AdS2

Jackiw-Teitelboim
AdS2 background

$$I_{JT} = -\frac{1}{16\pi G}\left[\int d^2x\phi\sqrt{g}(R+2) + 2\int_{bdy}\phi_b K\right]$$

Maldacena, Stanford, Yang, 1606.01857

Almheiri, Polchinski, 1402.6334

Quantum Chaos

Same pattern of
symmetry breaking

$$\langle V(a)W_3(b+\hat{u})V(0)W(\hat{u})\rangle \sim \frac{\beta\Delta^2}{C}e^{\frac{2\pi\hat{u}}{\beta}}$$

Schwarzian action

SYK dual to a quantum AdS2

Results

A feature of quantum chaos is RMT level statistics

Spectral Density of SYK-like models is doable analytically

RMT in the SYK?

Spectral density consistent with a gravity dual?

AGG, Verbaarschot Phys. Rev. D 94, 126010 (2016)   Phys. Rev. D 96, 066012 (2017)

A few weeks later Cotler et al.1611.04650

Are RMT features generic in quantum black holes?

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$

$$P(J_{ijkl}) = \sqrt{\frac{N^3}{12\pi J^2}} \exp\left(-\frac{N^3 J_{ijkl}^2}{12 J^2}\right)$$

Defining relation of a Euclidean N-dimensional Clifford algebra

Spectral density

Free energy: entropy, specific heat…… as expected

Level statistics

Spectral density

Combinatorial approach

Γ product of 4 χ matrices

$$M_{2p}(N) = \langle \mathrm{Tr}\, H^{2p} \rangle \qquad M_{2p} = \left\langle \mathrm{Tr} \sum \prod_{k=1}^{2p} J_{\alpha_k} \Gamma_{\alpha_k} \right\rangle$$

$N \to \infty$

$$M_{2p} = (2p-1)!! \langle J_\alpha^2 \rangle^p 2^{N/2}$$

Gaussian

Gaussian not related to a gravity dual!

Finite N corrections are important

Finite N

$$\Gamma_\alpha^2 = 1 \quad \Gamma_\alpha\Gamma_\beta - (-1)^r\Gamma_\beta\Gamma_\alpha = 0$$

$$M_{2p}(d) = \langle \mathrm{Tr} H^{2p} \rangle = \left\langle \mathrm{Tr}\left(\sum_\alpha J_\alpha \Gamma_\alpha\right)^{2p} \right\rangle$$

Suppression factor assuming no correlations

$$\eta_{N,q} = \binom{N}{q}^{-1} \sum_{r=0}^{q} (-1)^r \binom{q}{r}\binom{N-q}{q-r}$$

Number of crossings of a diagram

$$\alpha_P$$

$$Tr[\Gamma_\alpha\Gamma_\beta\Gamma_\gamma\Gamma_\alpha\Gamma_\beta\Gamma_\gamma]$$

$$\frac{M_{2p}}{M_2^p} = \sum_{\alpha_p} \eta_{N,q}^{\alpha_p} \quad ?$$

Riordan-Touchard formula!

J. Riordan, Mathematics of Computation 29, 215 (1975)

$$\frac{M_{2p}}{M_2^p} = \sum_{\alpha_p} \eta_{N,q}^{\alpha_p} = \frac{1}{(1-\eta_{N,q})^p} \sum_{k=-p}^{p} (-1)^k \eta_{N,q}^{k(k-1)/2} \binom{2p}{p+k}$$

Spectral density for Q(=η)-Hermite polynomials

Erdos, Mathematical Physics, Analysis and Geometry 17, 9164 (2014) $q \propto N^{1/2}$

Renjie Feng et al. 1801.10073 1611.04650

$$\rho(E) = \rho_{\rm QH}(E) = c_N \sqrt{1-(E/E_0)^2} \prod_{k=1}^{\infty} \left[1 - 4\frac{E^2}{E_0^2} \left(\frac{1}{2+\eta^k+\eta^{-k}}\right)\right]$$

How good is the n-correlation approx?

More precisely……


AGG, Jia, Verbaarschot, arXiv:1801.02696

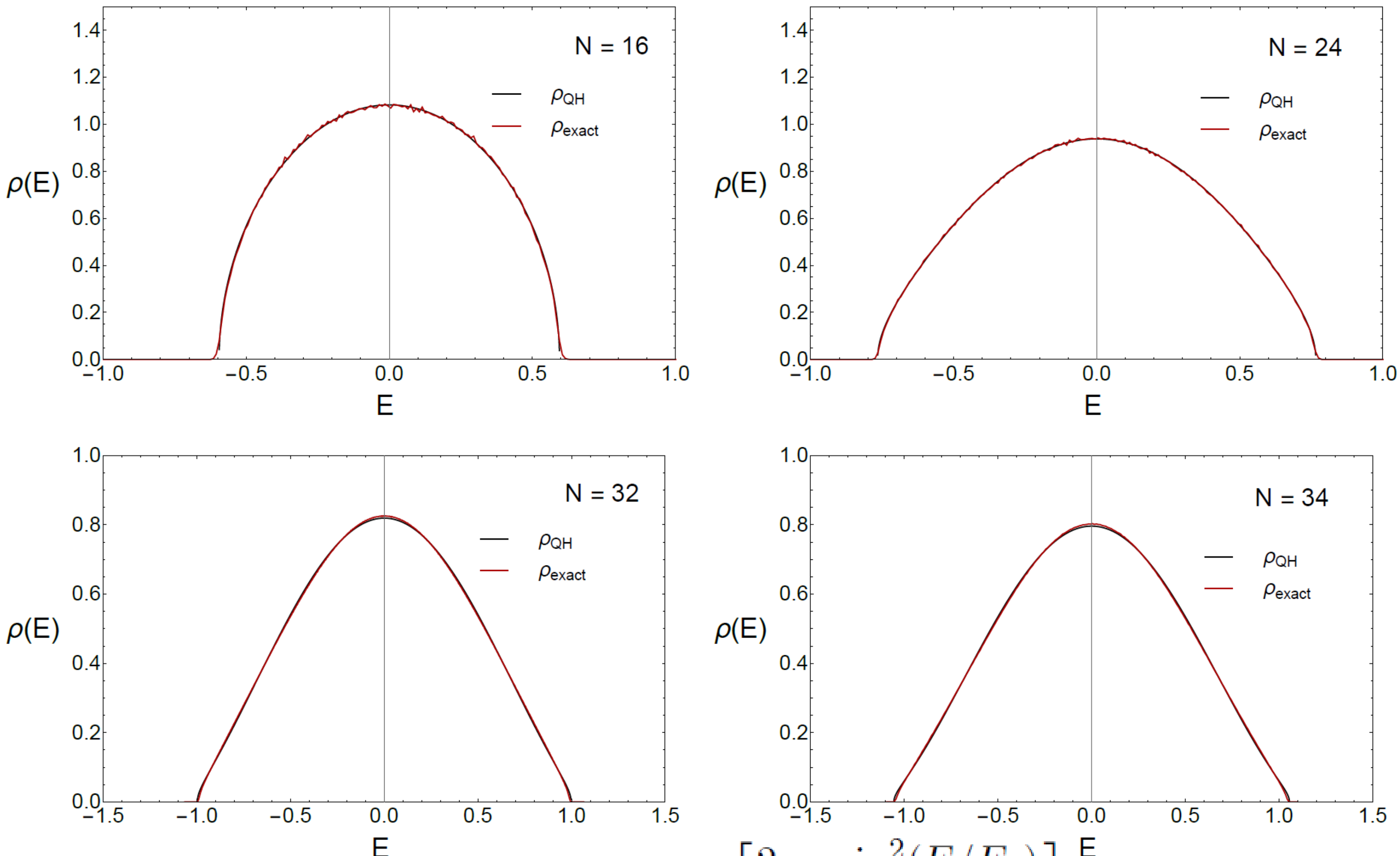
Q-Hermite gives the exact 1/N corrections

$1/N^2$ corrections can be computed explicitly

$$\frac{M_{2p}}{M_2^p} = \frac{1}{(1-\eta)^p} \sum_{k=-p}^{p} (-1)^k \eta^{k(k-1)/2} \binom{2p}{p+k} + \binom{p}{3} \left(\frac{8q^3}{N^2}\right)$$

$1/N^3$ and higher feasible Jia, Verbaarschot 1806.03271

Spectral correlations still Q-Hermite?



$$\rho_{\text{asym}}(E) = c_N \exp\left[\frac{2\arcsin^2(E/E_0)}{\log \eta}\right]$$

No fitting parameters !!!!

Looking at the tail

$$\rho_{\text{edge}}(E) \approx 2c_N \exp\left[\frac{\pi^2}{2\log\eta}\right] \sinh\left[\frac{2\pi\sqrt{2}\sqrt{1-(E/E_0)}}{-\log\eta}\right]$$

In agreement with solution from Schwarzian action

Bagrets, et al., 1702.08902

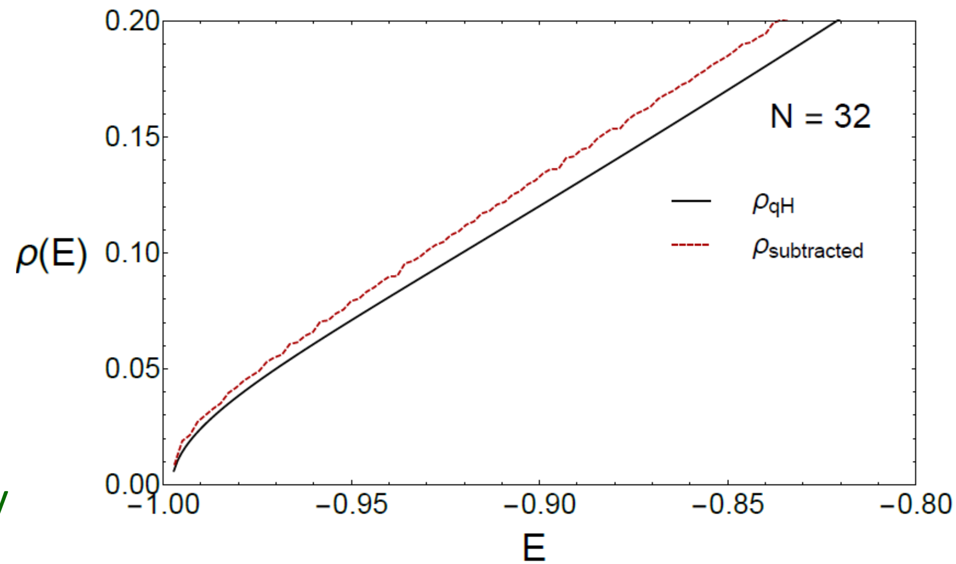
Stanford, Witten 1703.04612

Exponential increase

Cardy's formula

Bethe's formula

Black holes density



Sqrt edge

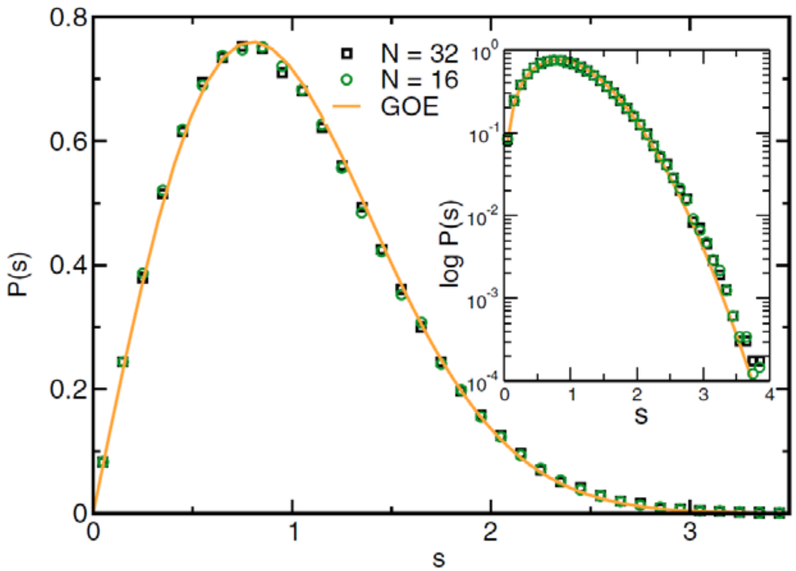
Typical of random matrices

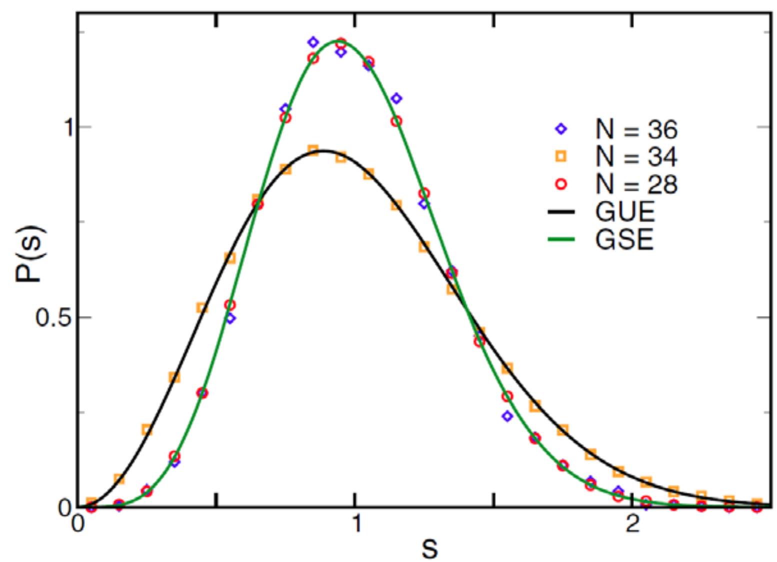
Bulk Level statistics

Level spacing distribution

$$P(s) = \sum_i \langle \delta(s - \epsilon_i + \epsilon_{i+1}) \rangle \quad \epsilon_i = E_i/\Delta$$

$$P(s) \approx a_\beta s^\beta \exp(-b_\beta s^2) \qquad \beta = 1\ GOE \qquad \beta = 2\ GUE \qquad \beta = 4\ GSE$$





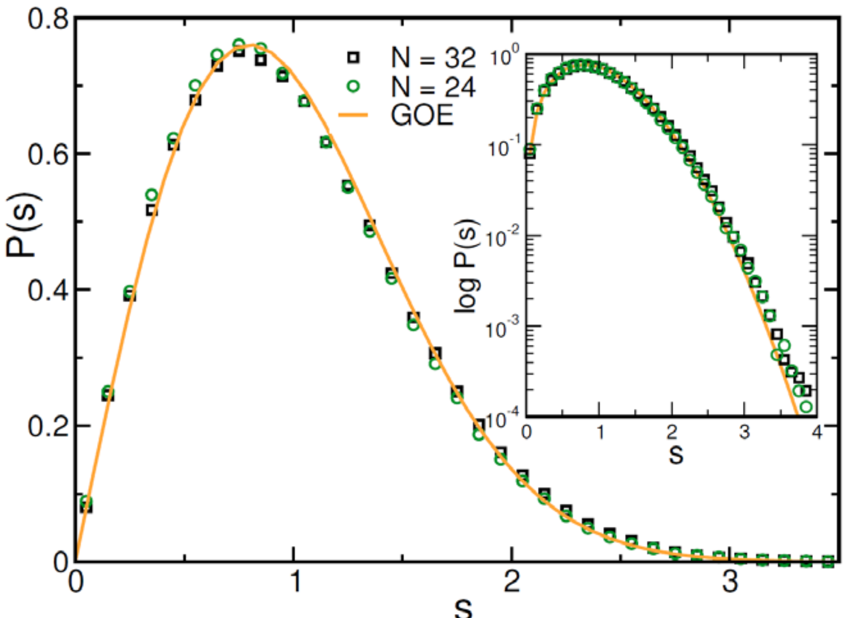
N dependent universality class
Why? Clifford algebra representations in N dimensions

You, Ludwig, Xu
1604.06964

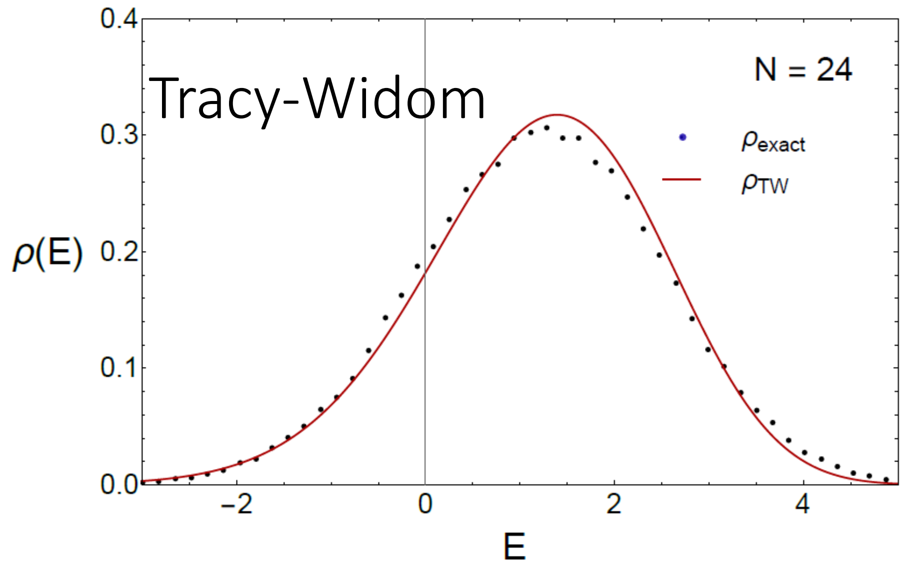
Level statistics close to the edge

Exponential increase of the density

Level spacing distribution



Distribution lowest eigenvalue



Still agreement with random matrix theory !!

Random matrix correlations characterize quantum black holes

Tenfold way in black hole physics?

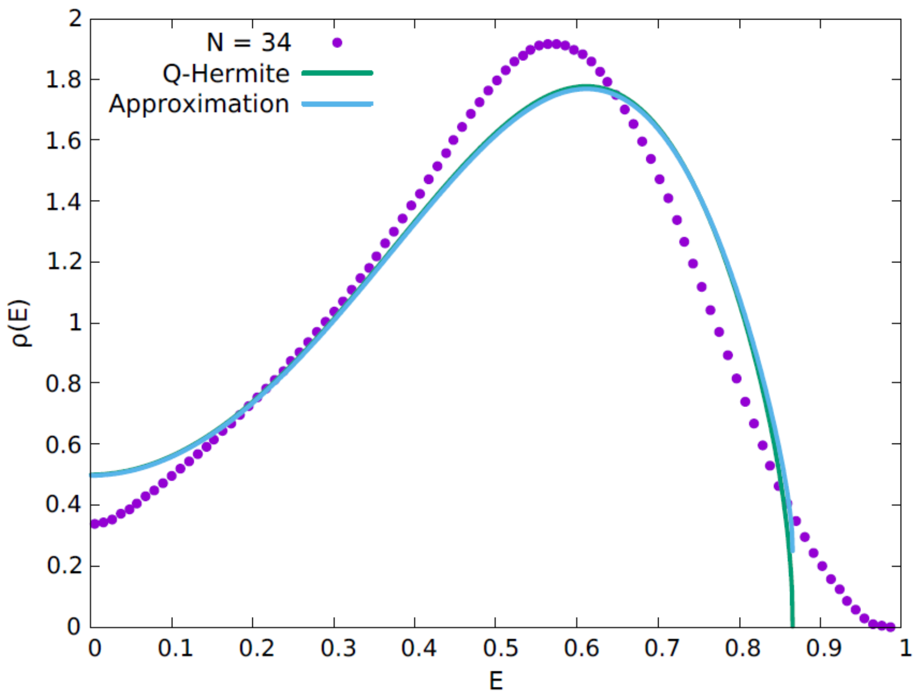
Yes!

Universality and Thouless energy in the supersymmetric SYK Model AGG, Jia, Verbaarschot, 1801.01071

1610.08917
Fu, Gaiotto, Maldacena, Sachdev

1702.01738
Li, Liu, Xin, Zhou

$$H = Q^2 \qquad Q = i \sum_{i,j,k=1}^{N} J_{ijk}\, \gamma_i\, \gamma_j\, \gamma_k$$



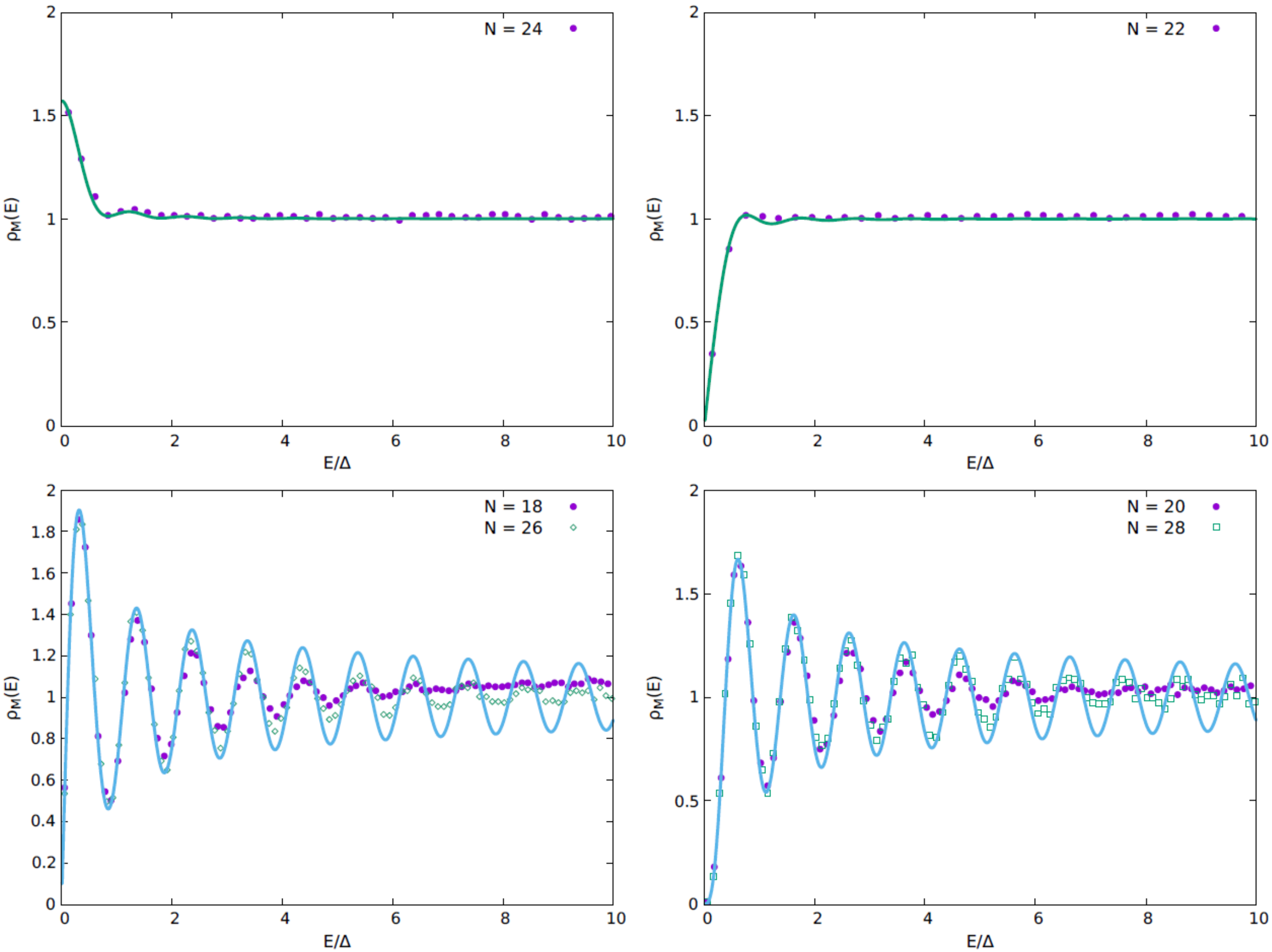
$$\rho_{\rm asym}(E) = c_N \cosh\left(\frac{\pi \arcsin(E/E_0)}{\log|\eta|}\right) \exp\left[2\frac{\arcsin^2(E/E_0)}{\log|\eta|}\right]$$

Q-Hermite analytical prediction just OK

Why?

Microscopic spectral density

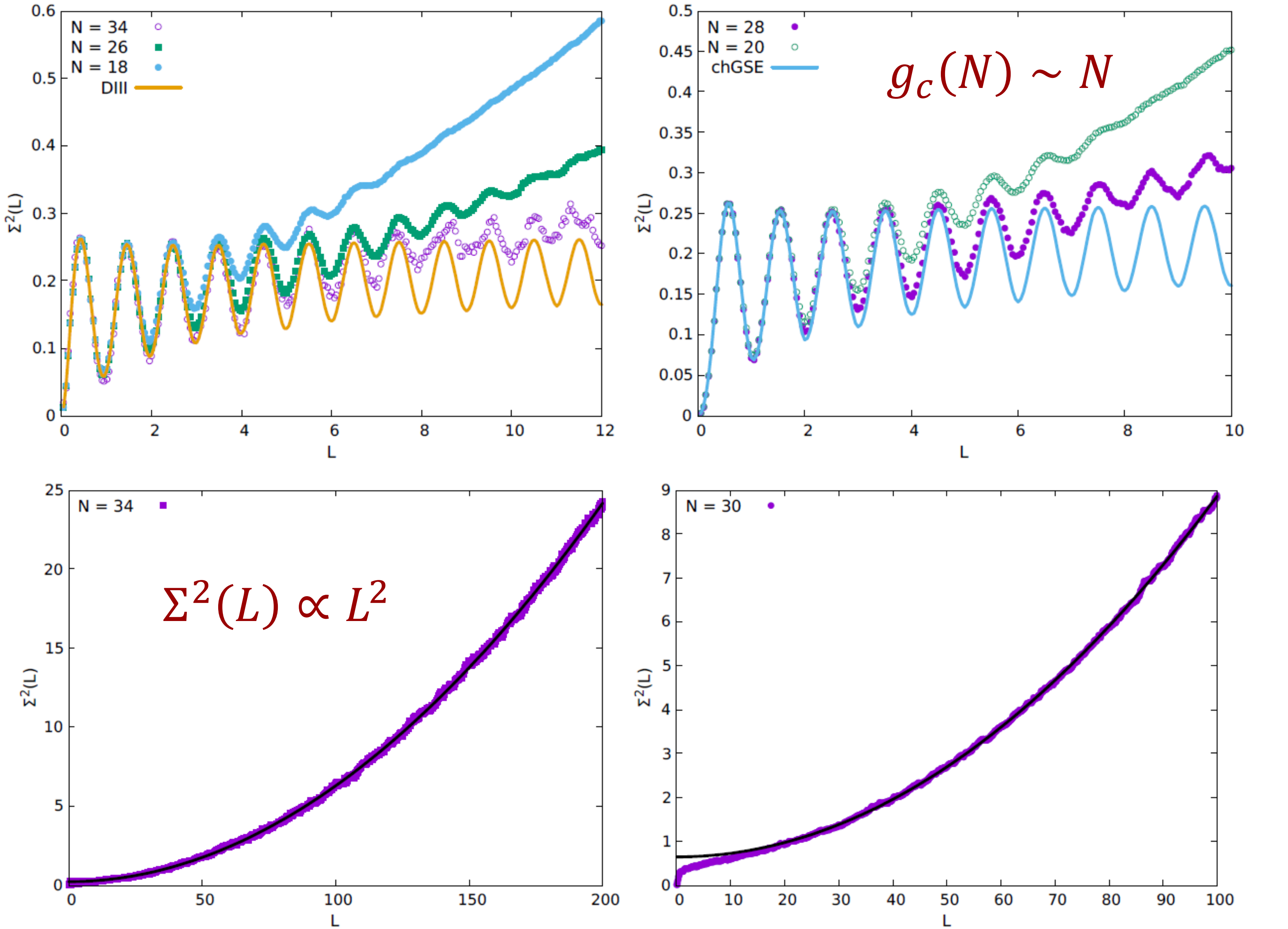
$$\rho_M(E) = \Delta \rho \left(\frac{E}{\Delta} \right)$$



Agreement with random matrix theory

Forrester, Verbaarschot…

Number Variance & Thouless Energy

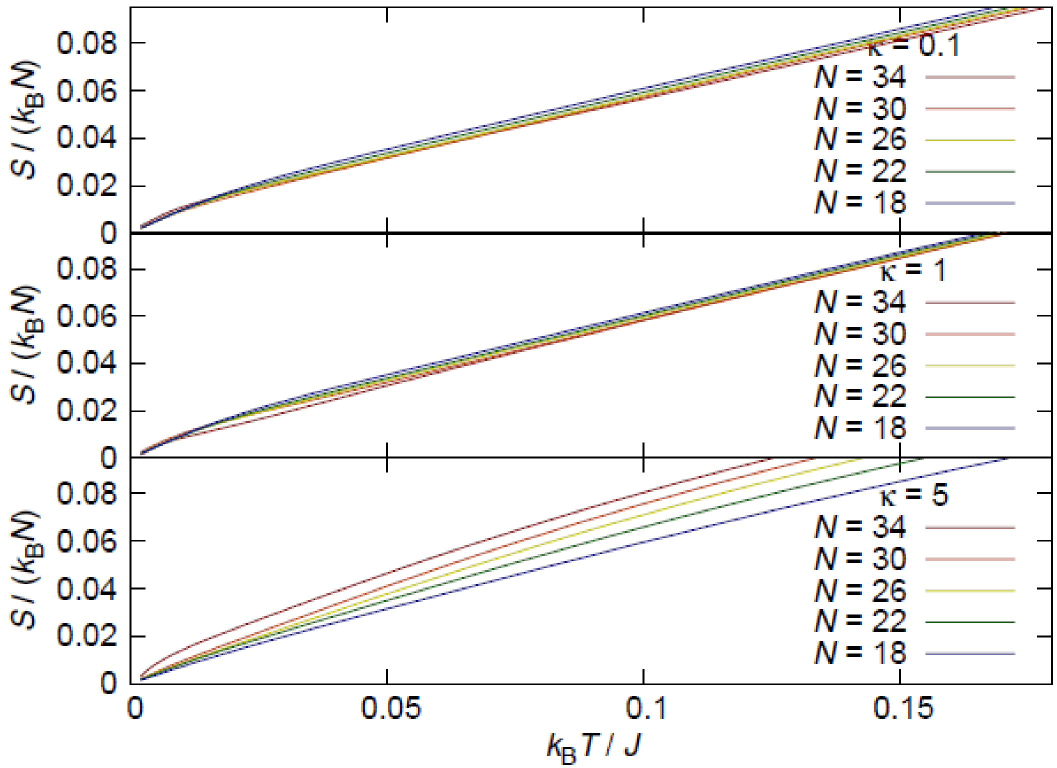


Chaotic-Integrable transition in the SYK model

A. Bermudez, AGG, B. Loureiro, M. Tezuka, PRL 120, 241603 (2018)

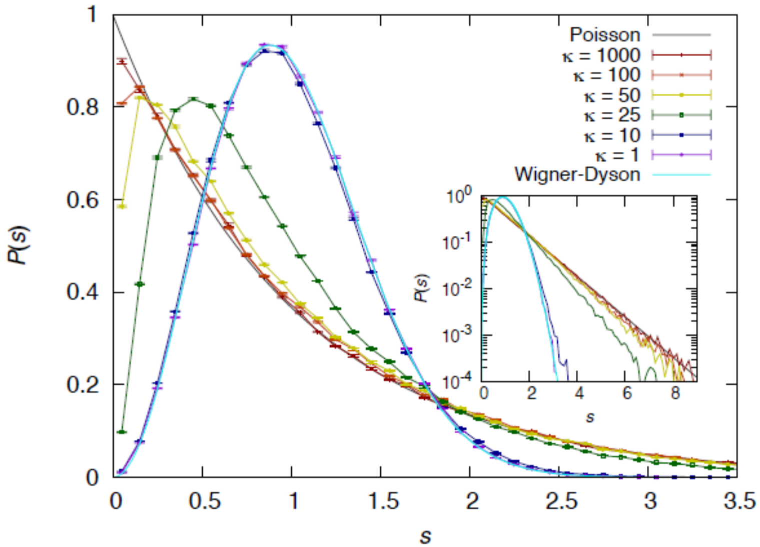
$$H = \frac{\kappa}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l + \frac{i}{2!}\sum_{i,j=1}^{N} K_{ij}\chi_i\chi_j$$

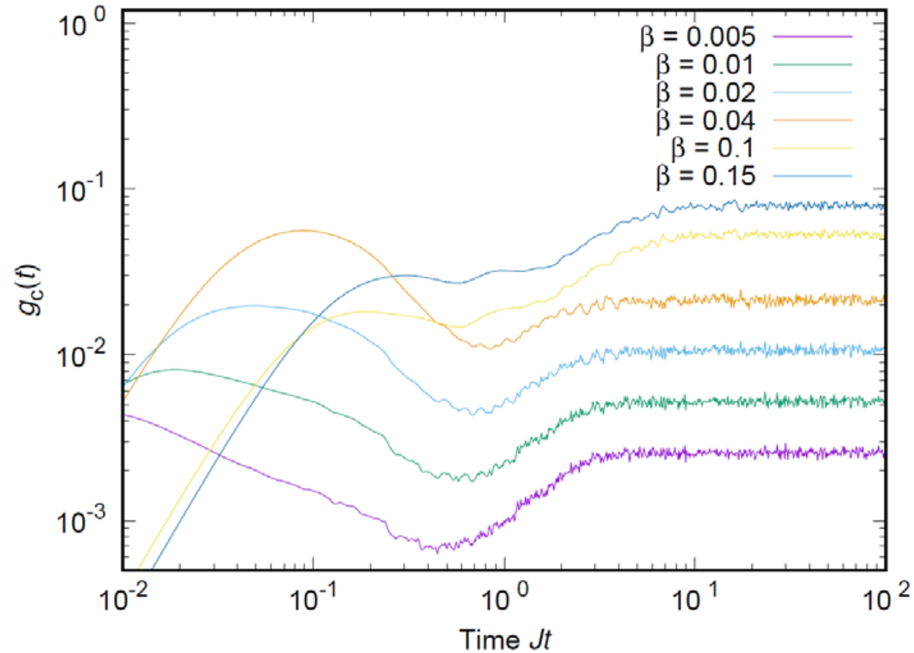
See also, Chen et al., PRL119, 207603 (2017)

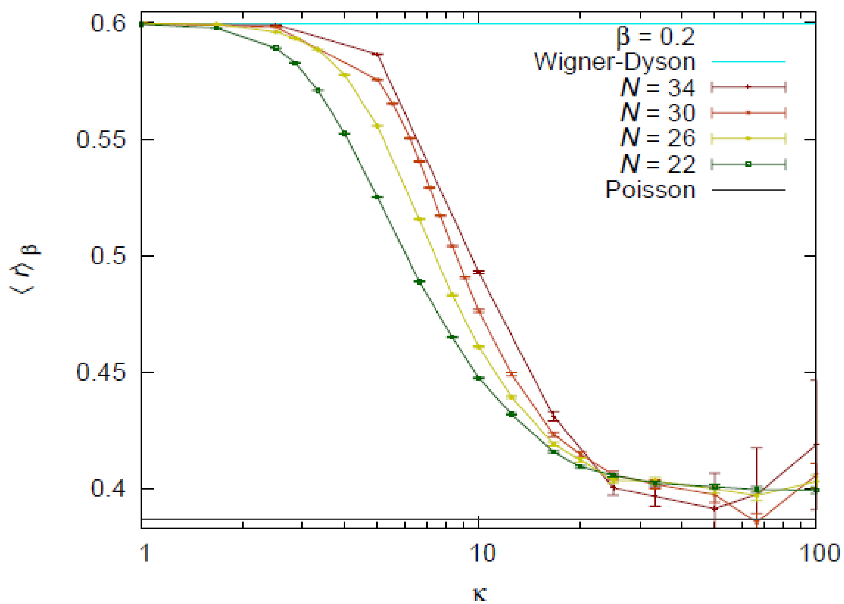


$$S_0 = 0$$

$$C_v = \mathrm{c}T$$
$$\mathrm{c} \propto N$$





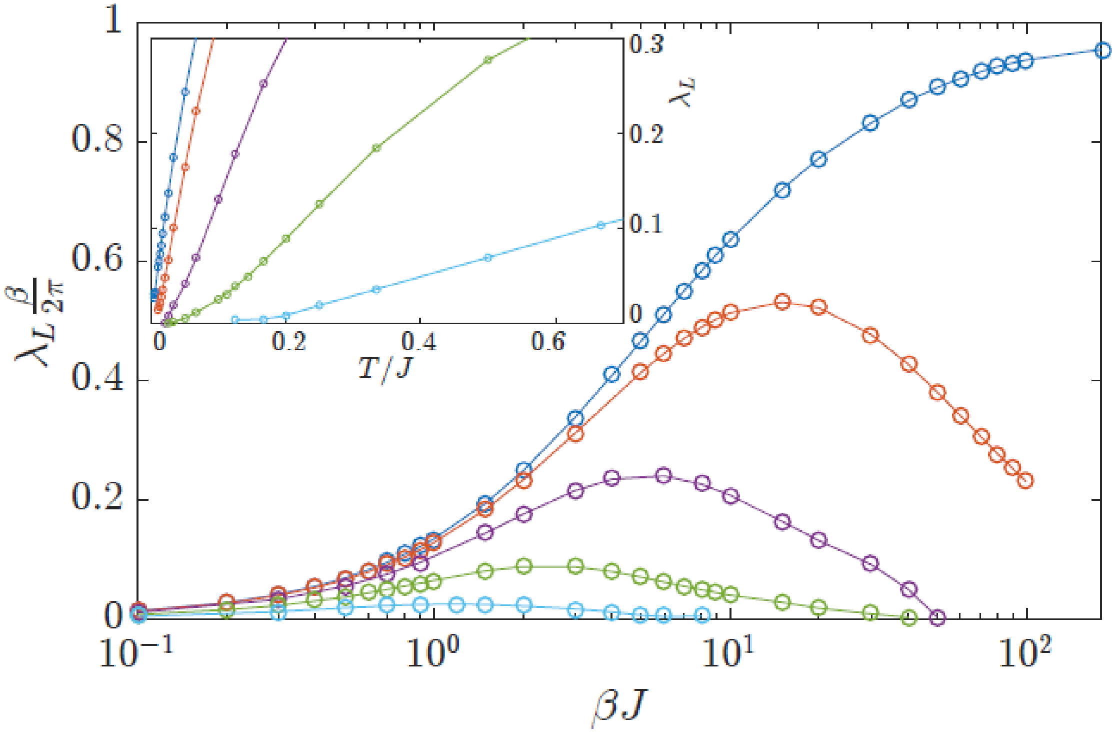


$$r_i = \frac{\min(\delta_i, \delta_{i+1})}{\max(\delta_i, \delta_{i+1})}$$

$$g(t,\beta) \equiv \left\langle \frac{Z(t,\beta)Z^*(t,\beta)}{Z(0,\beta)^2} \right\rangle$$

$$Z(t,\beta) = \mathrm{Tr}e^{-\beta H - iHt}$$

Chaotic – Integrable transition at $\kappa = \kappa_c$



Finite Lyapunov exponent only for high temperature

Chaos only for not too low T or not too strong coupling

Gravity dual?

Many body localization in the SYK model

AGG, Tezuka1801.03204

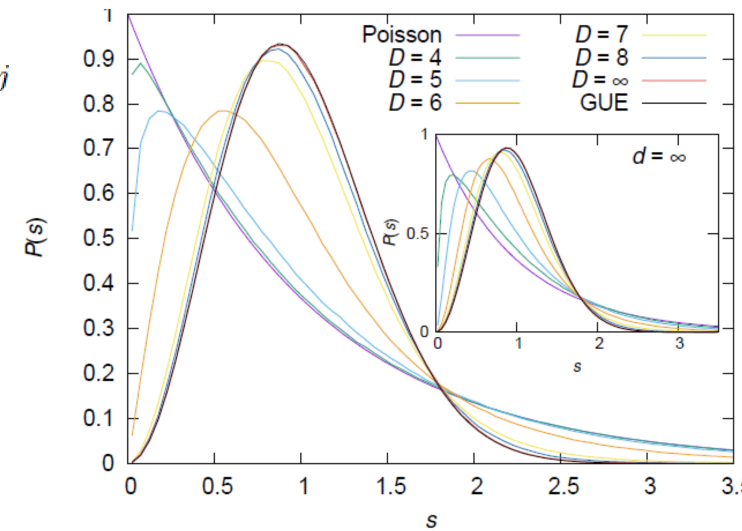
$$H = \sum_{1=i<j<k<l}^{N} \tilde{J}_{ijkl}(D)\,\chi_i\,\chi_j\,\chi_k\,\chi_l + i\kappa \sum_{1=i<j}^{N} \tilde{K}_{ij}(d)\,\chi_i\,\chi_j$$

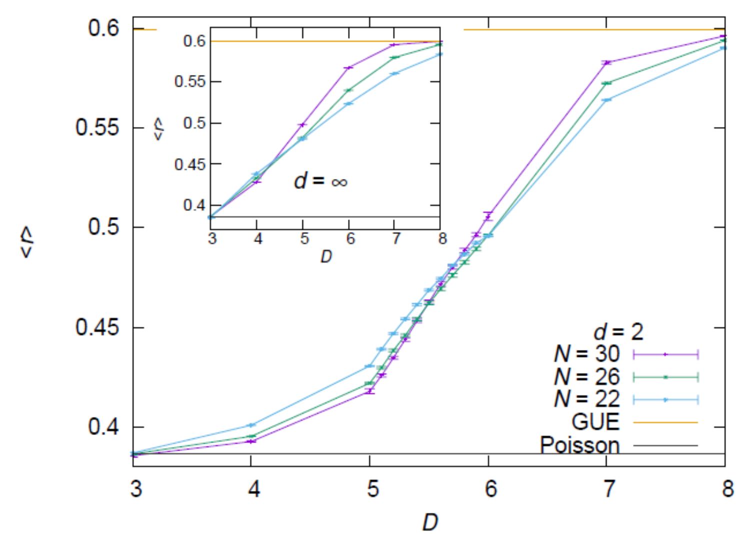
Reduction of the range interaction D

Many body metal-insulator transition

Different from Jian, Yao PRL 119, 206602 (2017)

What type of transition?





$$P(s) \sim e^{-As} \; A > 1 \; s \gg 1$$

$$\Sigma^2(L) \sim \chi L \;\; \chi < 1 \; L \gg 1$$

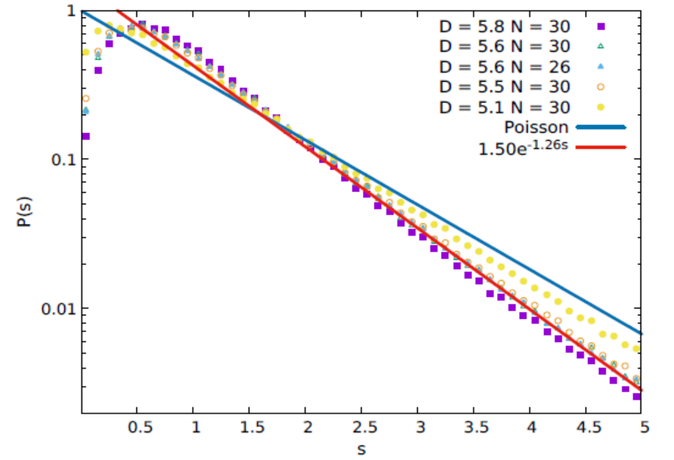
No correlation hole (dip)

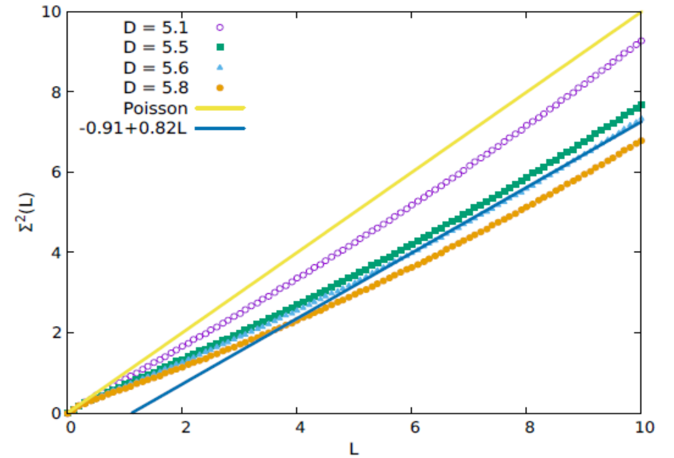
Coherence and interactions both important!

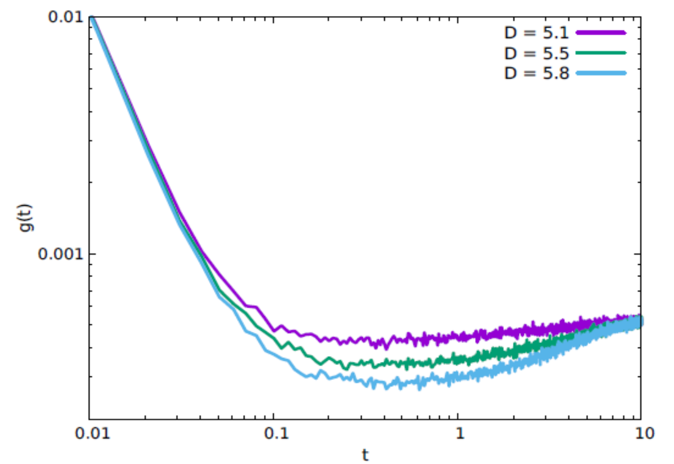
MBL transition in SYK

Gravity dual?

Analytical MBL transition?







Conclusions

Ergodicity and random matrix behaviour seems to be distinctive features of quantum black holes and their field theory duals

Quantum black holes may be classified according to random matrix theory

Generalized SYK models undergoing metal-insulator and chaotic-integrable transitions open new research avenues in both condensed matter and high energy

Thanks!

Low Temperature
Strong coupling

$$S = -N\frac{\alpha_S}{\mathcal{J}}\int d\tau\{f,\tau\}$$

$$-\beta F \supset \frac{N\alpha_S}{\mathcal{J}}\int_0^\beta d\tau\left\{\tan\frac{\pi\tau}{\beta},\tau\right\} = 2\pi^2\alpha_S\frac{N}{\beta\mathcal{J}}$$

OTOC:

$$\frac{\langle\psi_i(0)\psi_j(\tau)\psi_i(0)\psi_j(\tau)\rangle}{\langle\psi_i(0)\psi_i(0)\rangle\langle\psi_j(\tau)\psi_j(\tau)\rangle} \propto 1 + i\frac{\beta J}{N}e^{\frac{2\pi\tau}{\beta}}$$

Linear Specific Heat

Exponential Growth of OTOC

Same pattern of symmetry breaking

SYK
dual
Quantum AdS2

Why is SYK interesting?

Toy model of quantum gravity

"Solvable" for large but finite N

Explicit 2-pt, 4-pt calculations

Emergent conformal symmetry in the IR

Explicitly and spontaneously broken but weakly
Same as in AdS_2 gravity backgrounds
Exponential growth of the spectral density

Maximally chaotic

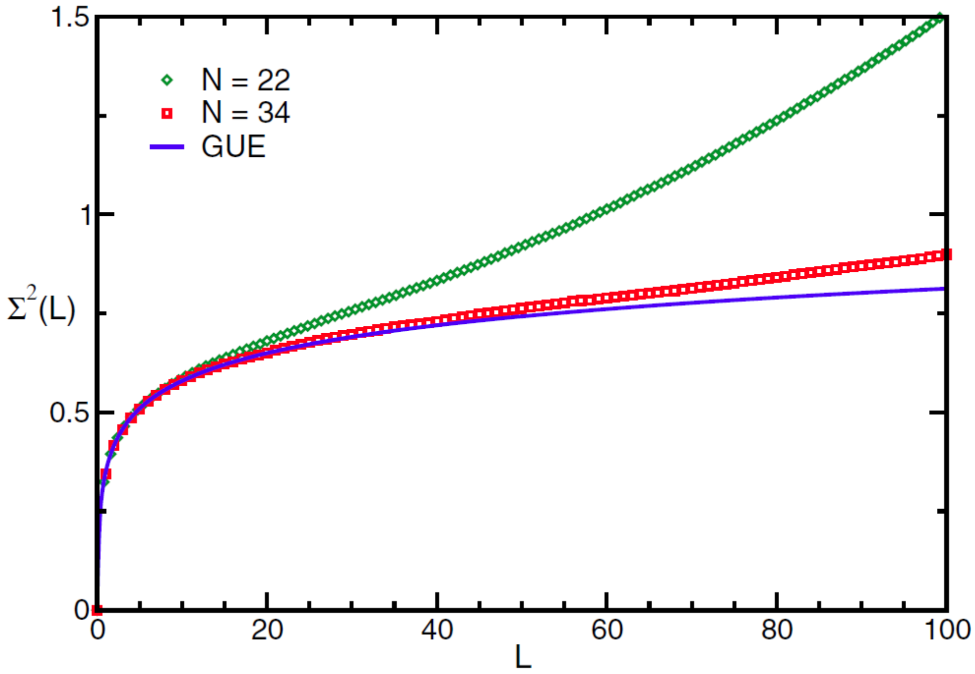
Lyapunov exponent as in black-holes that saturates the Maldacena-Shenker-Stanford bound on chaos

Remarks on the Sachdev-Ye-Kitaev model
J. Maldacena, D. Stanford, Phys. Rev. D 94, 106002 (2016)

Thouless Energy in the SYK model

Number variance $\Sigma^2(L) = \langle N^2(L) \rangle - \langle N(L) \rangle^2$

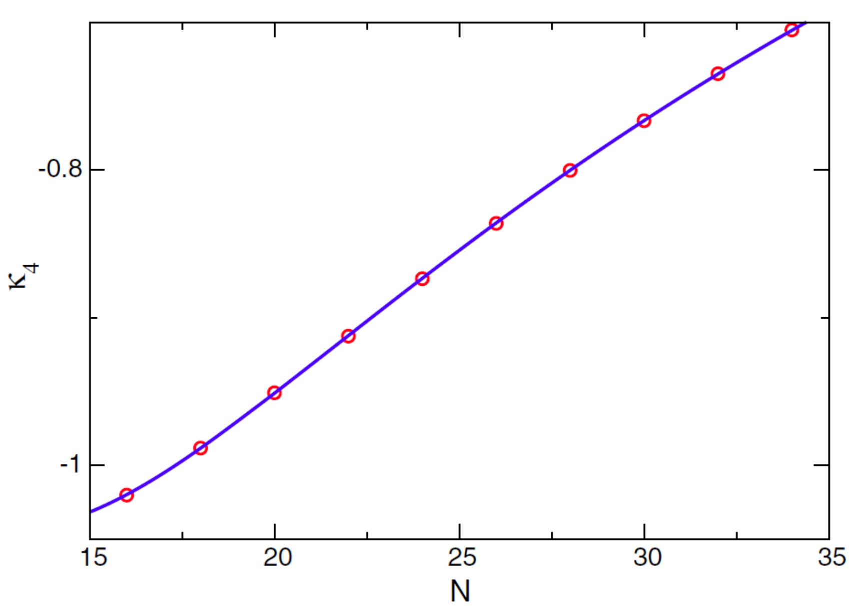
GUE $\Sigma^2(L) \approx c_\beta(\log(d_\beta \pi L) + \gamma + 1 + e_\beta \ldots)$

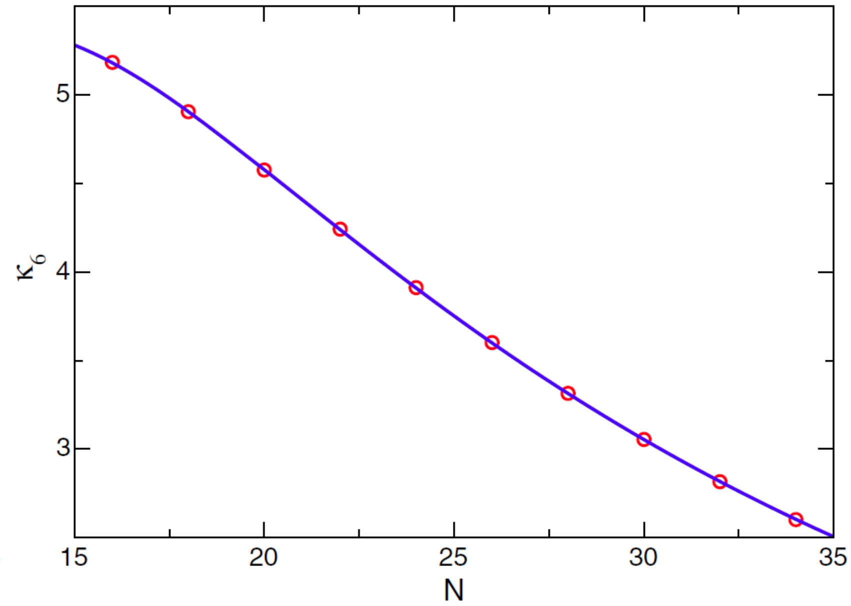


$\Sigma^2(L) \propto L^2$

$L \gg 1$

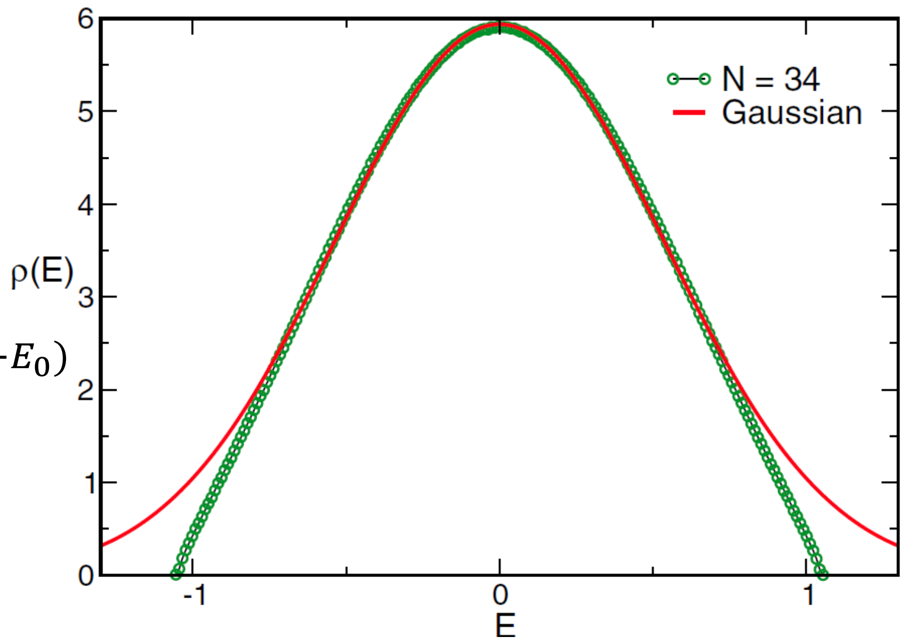
$g_c(N)$?





$$\rho(E) \sim e^{a\sqrt{(E-E_0)}}$$

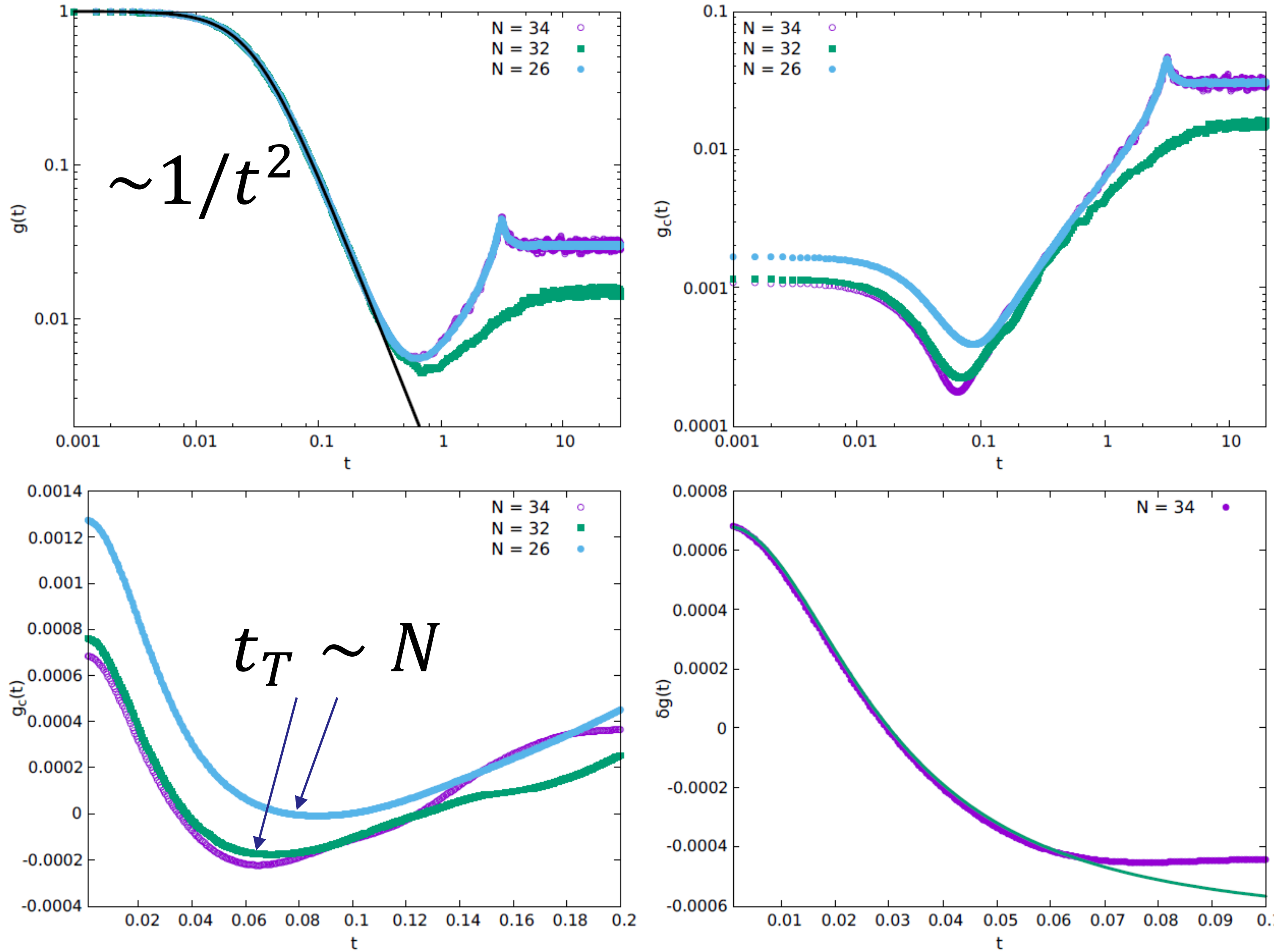
?



Can we do better?

Yes

Spectral form factor



$$\delta g(t) = g(0)\frac{\beta^2 - t^2}{\beta^2 + t^2}$$

Kitaev 2015 Also: 1711.0847

"A simple model of quantum holography"

http://online.kitp.ucsb.edu/online/entangled15/kitaev/

$$H = J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$

Strong coupling

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

$$\beta J \gg 1$$

$$\tau J \gg 1$$

$$\langle J_{ijkl}^2 \rangle = \mathrm{J}^2/\mathrm{N}^3$$

AdS2

Quantum gravity?

SYK = Sachdev-Ye-**Kitaev**

Random Matrix

Dyson-Mehta

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}$$

$\beta = 1$ GOE

$\beta = 2$ GUE

$\beta = 4$ GSE

Semicircle law $\rho(E) \sim \sqrt{E_0^2 - E^2}$ No universal

Level Repulsion $P(s) \sim s^{\beta} e^{-As^2}$ $s = (E_{i+1} - E_i)/\Delta$

Spectral rigidity

$$\Sigma^2(N) = \langle n(N)^2 \rangle - \langle n(N) \rangle^2 \sim \log(N)$$

Universality: Quantum Chaos, Mesoscopic physics….

Universality class depends on N

N	$(C_1K)^2$	$(C_2K)^2$	C_1KC_2K	RMT
2	1	-1	$-i\Gamma_5$	GUE
4	-1	-1	$-\Gamma_5$	GSE
6	-1	1	$-i\Gamma_5$	GUE
8	1	1	Γ_5	GOE
10	1	-1	$-i\Gamma_5$	GUE
12	-1	-1	Γ_5	GSE

Why? Clifford algebra representations in N dimensions

You, Ludwig, Xu
1604.06964

Bulk level statistics is well described by random matrix theory

Weak N dependence of short-range spectral correlators

SYK is ergodic and always thermalizes for high energy initial states

Correction to random matrix, low energy?

Outline

1. Models with infinite range interactions before SYK, random matrix theory and quantum chaos

2. An introduction to the SYK model

3. SYK model, black holes, random matrices and chaotic-integrable transitions

Butterfly effect

Classical chaos

Hadamard 1898

Lyapunov 1892

$$\|\delta x(t)\| = e^{\lambda t}\|\delta x(0)\|$$

$$\lambda > 0$$

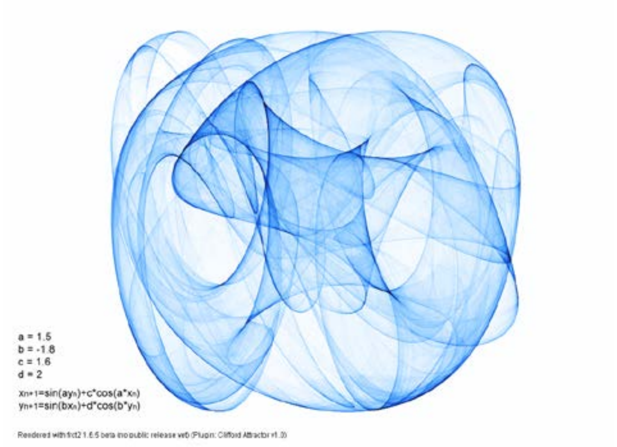
$$h_{KS} > 0$$

Pesin theorem

Difficult to compute!

Lorenz 60's

Meteorology

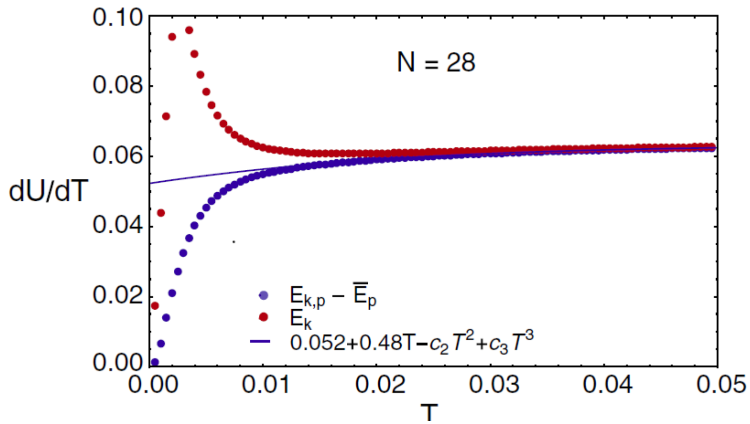


Thermodynamic properties

$$-\beta F \supset -\sum_{n=2}^{\infty} \log \frac{n}{\beta J} + \text{const} \to \#\beta J - \frac{3}{2}\log \beta J + \text{const}$$

$$\frac{dU(T)}{dT} = \frac{q(N)}{N} + c(N)T + c_2(N)T^2 + c_3(N)T^3$$

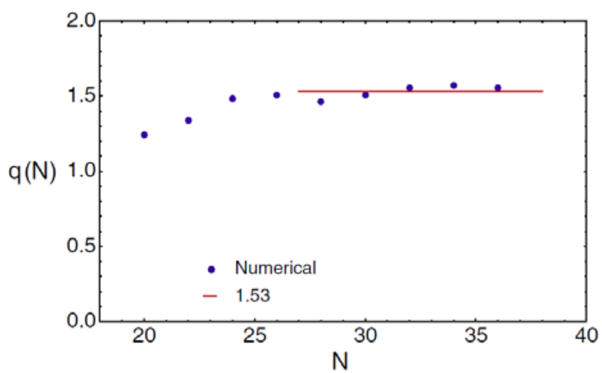
2-body interactions

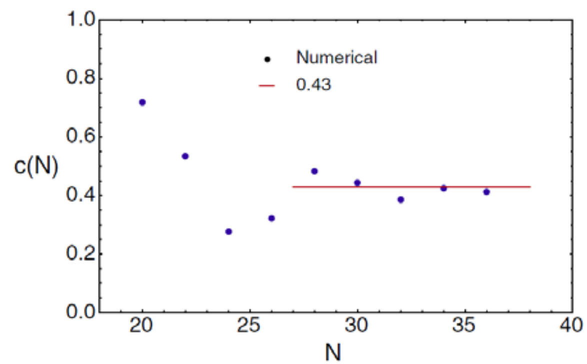


See also:

Jevicki et al., arXiv:1608.07567

Cotler et al. arXiv:1611.04650





$q = 1.53 \pm 0.2,$ $\quad c/N = 0.43 \pm 0.10$ $\quad S_0 = 0.21N$

q = 3/2 $\quad$ c/N = 0.40 $\quad$ $S_0 = 0.23N$

Reasonably good agreement with large N predictions

Corrections

Classical

$$\frac{1}{N} \ll 1$$

Conformal

$$\frac{1}{J\beta} \ll 1$$

Why?

Thermodynamic properties

(Quantum) chaos bound

In the conformal limit:

Reparametrization invariance

$$G \longrightarrow G_f(\tau,\tau') = [f'(\tau)f'(\tau')]^{\Delta} G(f(\tau), f(\tau'))$$

Conformal symmetry spontaneously broken

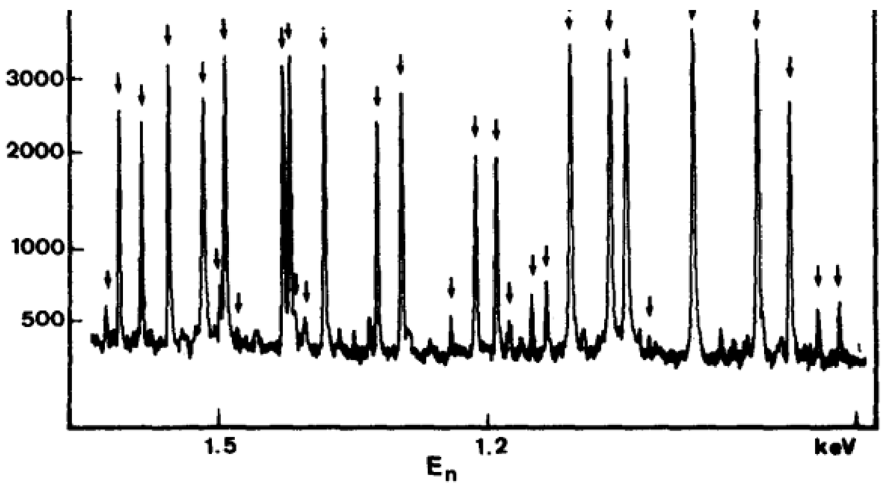
f Goldstone modes

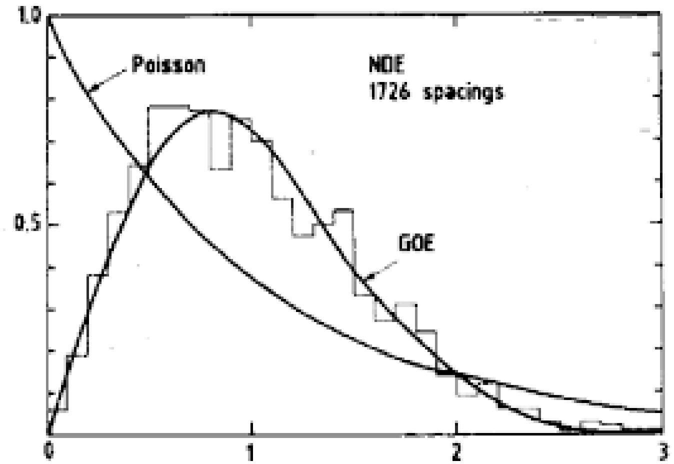
Introduction to the SYK model

Nuclear Physics 60's:

The ultimate approximation "A random matrix as an effective nuclear Hamiltonian"

Fermionic quantum dot with N-body random interactions of infinite range





O. Bohigas, R.U. Haq, and A. Pandey, in Nuclear Data for Science and Technology, (1983)

Coceva and Stefanon, Nuclear Physics A, 1979

Flores, Bohigas, French 1970

Quantum Chaos in holography