FAKULTÄT FÜR PHYSIK

UNIVERSITÄT DUISBURG ESSEN

Collective and Incoherent Single–Particle Motion in Interacting Many–Body Systems

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Random Matrices, Integrability and Complex Systems

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Outline

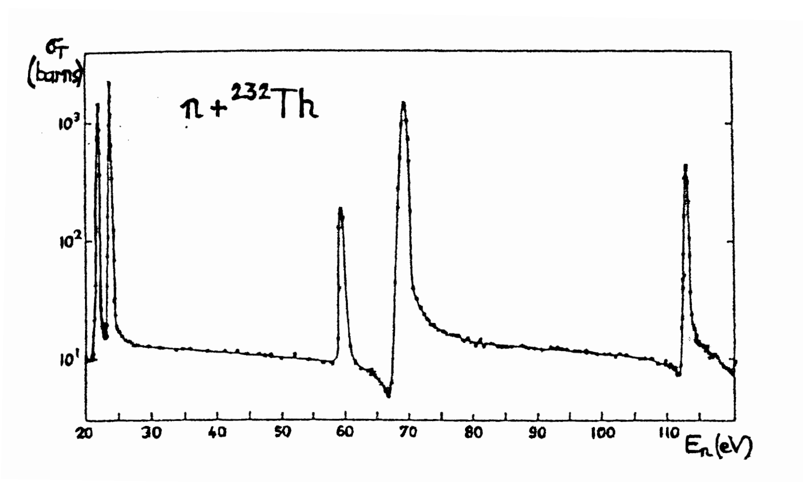
- brief review of chaos in nuclei
- what is the role of collective excitations for chaoticity ?
- first step: toy model of coupled clouds/chains
- exact derivation of spreading from microscopic model
- second step: semiclassics for kicked spin chains
- simultaneously incoherent and collective motion
- spectrum can be dominated by collectivity

Introduction:

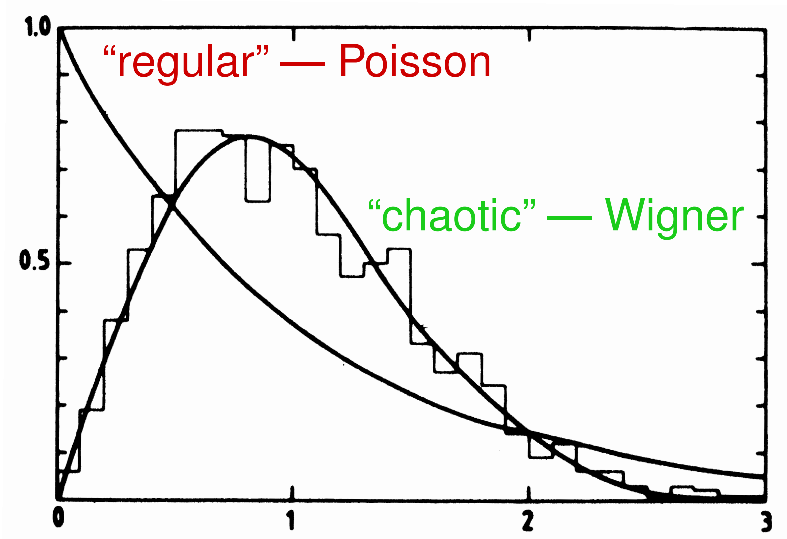
Single–Particle vs Many–Body Systems and Chaos

Quantum Chaos in Nuclei

nuclear data ensemble (far from groundstate)



resonances

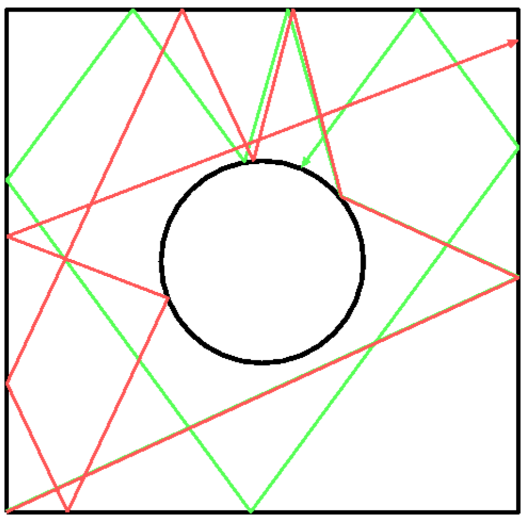


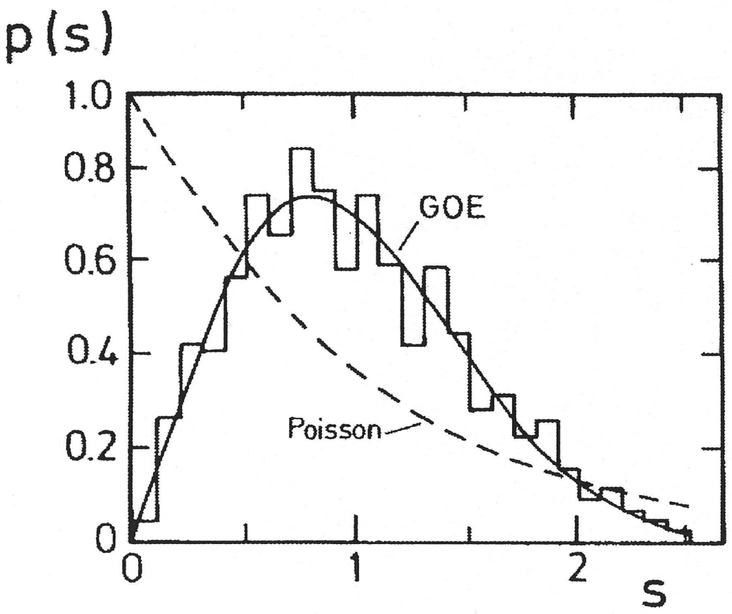
spacing distribution

$\longrightarrow$ quantum "chaos", random matrix statistics . . . but why ?

Haq, Pandey, Bohigas (1982)

Quantum Chaos in Single Particle Systems

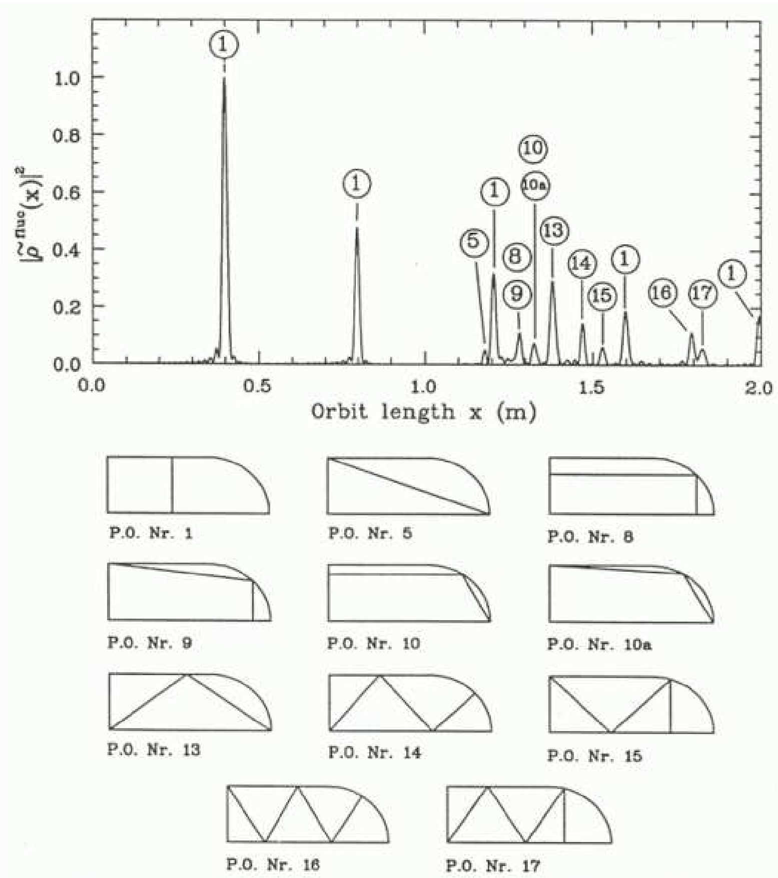




orbits separate at an exponential rate $\longrightarrow$ classical CHAOS

$\longrightarrow$ quantum spectrum shows random matrix statistics

Semiclassics and Periodic Orbits



periodic orbits of classical system yield quantum spectrum to $\mathcal{O}(\hbar)$

classical chaos implies random matrix statistics (better would be: quantum $\rightarrow$ classical)

Gutzwiller (1967), Berry (1985), Sieber, Richter (2001),

Haake, Heusler, Müller, Braun, Altland (2004–2007)

Gutzwiller Trace Formula

to leading order $\hbar$

$$\sum_n \delta(E - E_n) = \bar{\rho}(E) + \frac{1}{\hbar}\mathrm{Re}\sum_\gamma A_\gamma \exp\left(\frac{i}{\hbar}S_\gamma(E)\right)$$

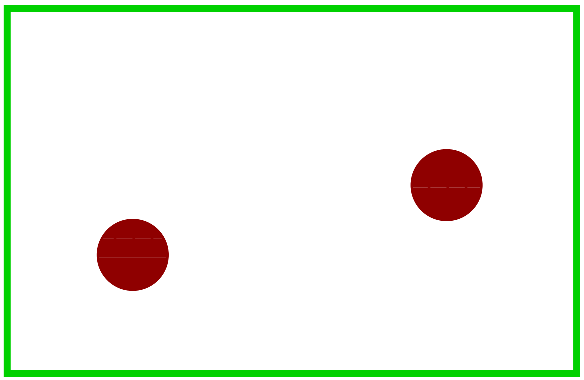
γ periodic orbit, S_γ action, A_γ stability

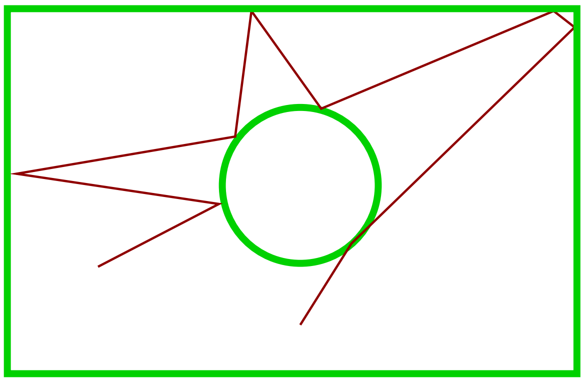
periodic orbits are the skeleton of the quantum spectrum

Gutzwiller (1967)

Chaos in Many–Body Systems

but nuclei consist of many particles! — let us begin with two:





two hard spheres in a box with hard walls and periodic boundary conditions (torus)

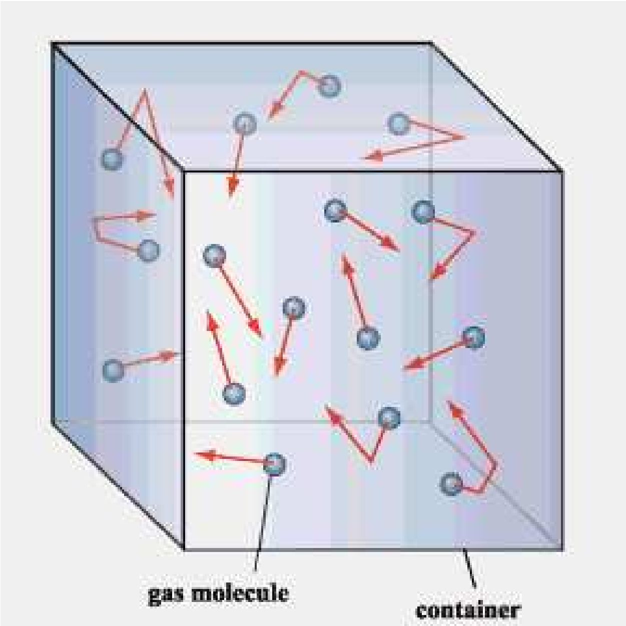
remove center of mass motion $\longrightarrow$ point particle in Sinai billiard

that two–body system is chaotic

Sinai (1963,1970)

Classical Boltzmann Gas

N hard spheres in a d dimensional box with hard walls



equivalent to highly complicated generalization of Sinai billiard

if particle density not too high: dynamics chaotic (hyperbolic, i.e., positive Lyaponow exponents) and ergodic

Simanyi (2003–2010)

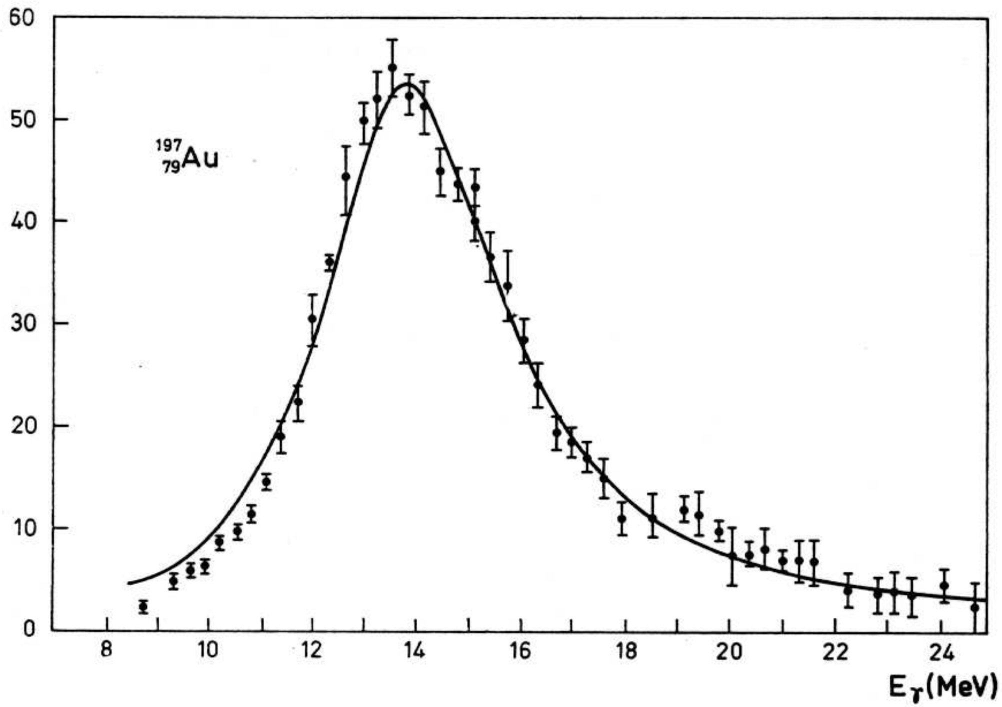
Where is the Problem?

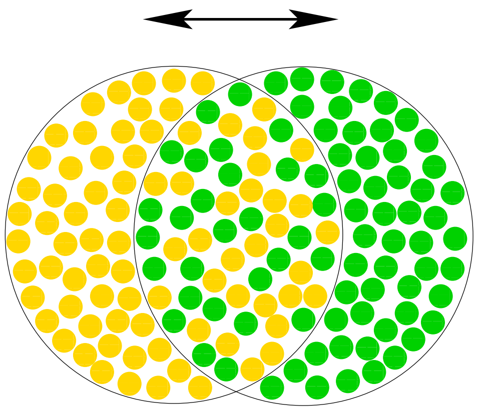
- little known about quantum Boltzmann gas which would be similar to nucleus
- in contrast to Boltzmann gas, nuclei are selfbound
- selfboundness implies existence of maximum energy
- classical–quantum proof for single–particle chaos does not work for many particles because of collective motion, i.e., coherent motion in phase space

Collective Excitations in Nuclei

Example for Electric Giant Dipole Resonance

seen in many nuclei, here for Gold ($A = 197$)





high energies

Fultz, Bramblett, Caldwell, Kerr, PR 127 (1962) 1273

Strength Function

cross section contains huge number of individual states (fragmentation) which cannot be resolved

strength function is related to this: consider state $|\mathrm{GDR}\rangle$ with resonance energy E_{GDR} and couple many states to it
$\rightarrow$ density around $|\mathrm{GDR}\rangle$

$$\varrho_{\mathrm{GDR}}(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E - E_{\mathrm{GDR}})^2 + \Gamma^2/4}$$

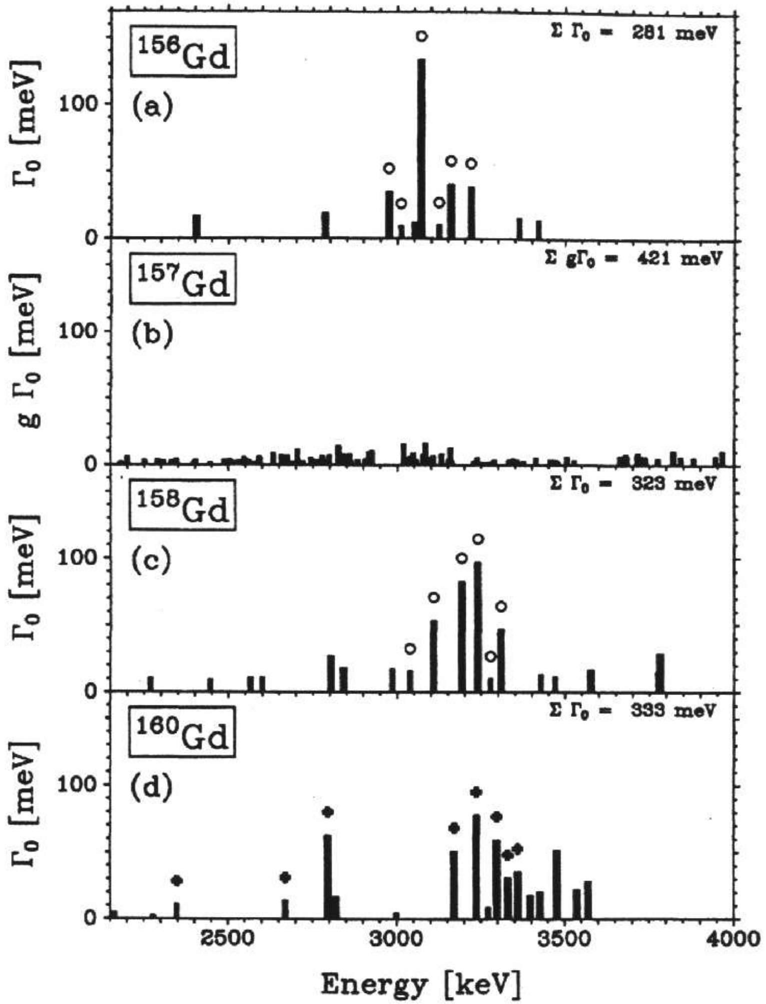
(Breit–Wigner)

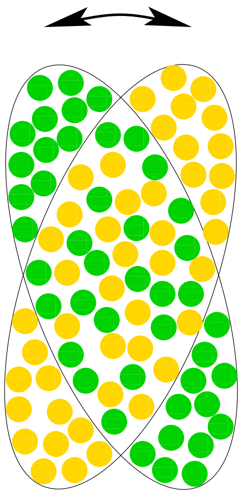
under rather general conditions!

$\longrightarrow$ strictly, one cannot conclude chaotic fluctuations, but at these GDR energies one certainly expects them

Bohr, Mottelson, Nuclear Structure (1969); Zelevinsky, Annu Rev Nucl Part Sci 46 (1996) 237

Scissors Mode Resonances in Gadolinium





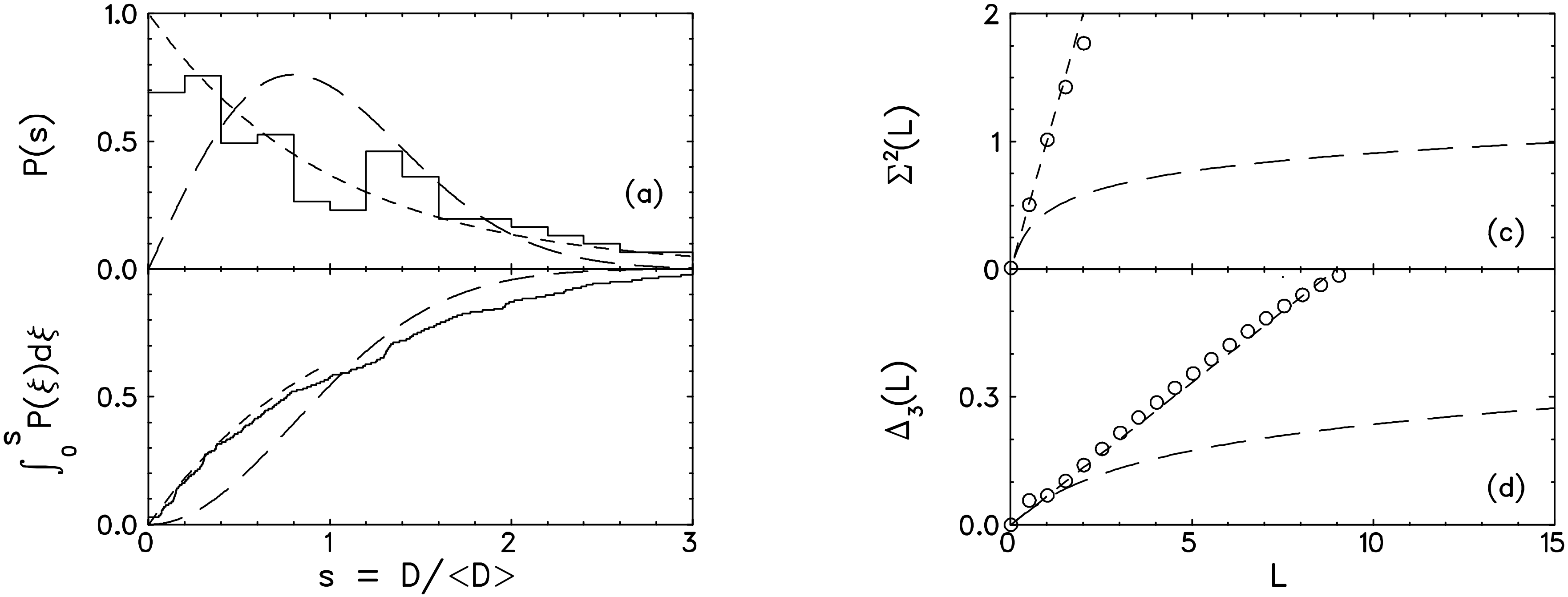
low energies

fragmentation resolved
in experiment

Margraf et al, PRC 52 (1995) 2429

Spectral Statistics of Scissors Mode Resonances

152 states in 13 heavy deformed nuclei (at least 8 per nucleus)

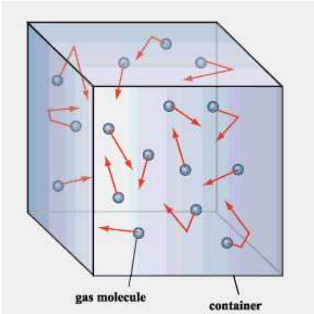


$\longrightarrow$ Poissonian (regular) behavior!

Enders, Guhr, Huxel, von Neumann–Cosel, Rangacharyulu, Richter, PLB 486 (2000) 273

The Role of Collective Motion

- sound is collective motion in classical Boltzmann gas



- little known about collective excitations in quantum Boltzmann gas
- they might “die out” as energy becomes very large
- but in nuclei selfboundness, maximum energy
- attempts to prove chaoticity of the classical dynamics in nuclei and, based on that, random matrix statistics have to cope with collective motion and its regularity

What needs to be done next?

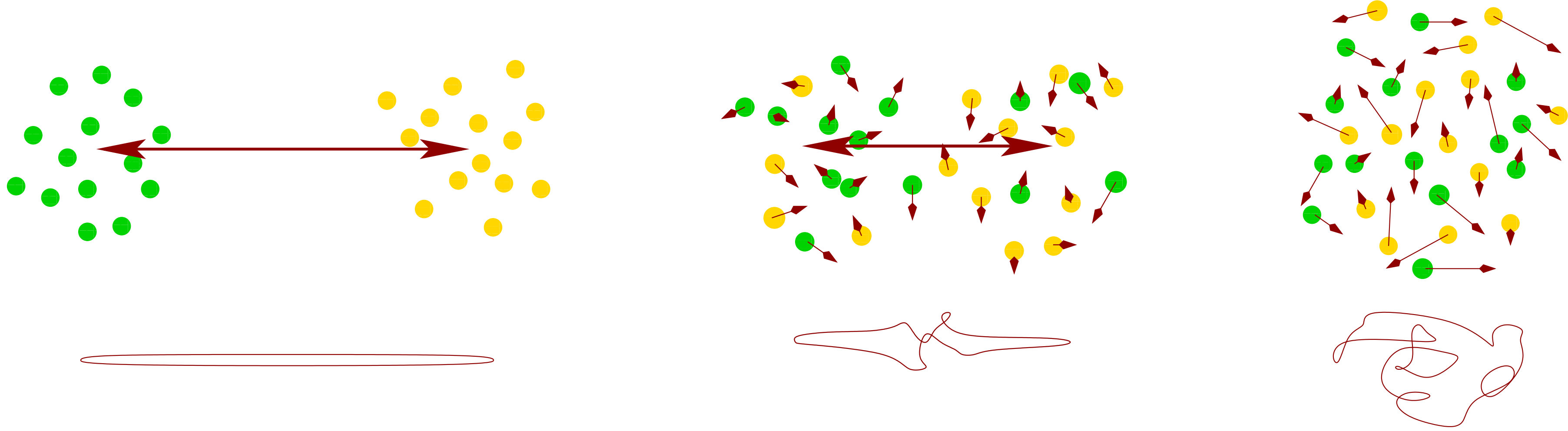
big goal: identify conditions for regular and chaotic motion in many–body systems

big goal: do that semiclassically and understand the periodic orbit structure

modest goal: study interplay between collective and incoherent degrees of freedom in analytically tractable non–effective, but microscopic toy model with full control over dynamics of original particles

Toy Model of Giant Dipole Resonance

Two Interacting Clouds



spreading, relates to relaxation of a system far from equilibrium

Coupled Chains of Oscillators in One Dimension

N particles in each cloud, positions $\vec{x}^{\,(l)} = (x_1^{(l)}, \ldots, x_N^{(l)}),\ l = 1, 2$

$$H^{(l)} = \frac{(\vec{p}^{\,(l)})^2}{2m} + \frac{1}{2}(\vec{x}^{\,(l)})^T W \vec{x}^{\,(l)}$$

interaction matrix W preserves translational invariance

full Hamiltonian $\quad H = H^{(1)} + H^{(2)} + V^{(c)}$

with coupling term $\quad V^{(c)} = \sum_{i,j} K_{ij}(x_i^{(1)} - x_j^{(2)})^2$

objection: integrable (recurrence time, no thermalization)
answer: time scales adjustable, N large, still spreading

Collective Coordinate and Diagonalization

collective coordinate is difference between centers of masses

$$X = \frac{1}{\sqrt{2N}} \sum_{i=1}^{N} x_i^{(1)} - \frac{1}{\sqrt{2N}} \sum_{i=1}^{N} x_i^{(2)}$$

diagonalization, $N-1$ orthogonal non–collective, interacting coordinates $\xi_i = 1, \ldots, N-1$ of oscillators coupled to X

reminiscent of, but different from Caldeira–Leggett model: always finite, $N < \infty$, “bath” is part of the system

Equation of Motion — Quantum and Classical

Heisenberg equation for quantum case

$$\frac{d^2\hat{X}(t)}{dt^2} + \Omega^2\hat{X}(t) + \int_0^t ds\ \dot{\gamma}(t-s)\hat{X}(s) \ = \ \frac{1}{m}\hat{F}$$

spreading (not damping) kernel $\gamma(t)$
and force $\hat{F}$ due to coupling to “bath”

$\longrightarrow$ equation for $\langle\hat{X}(t)\rangle$ which coincides with equation for collective coordinate $X(t)$ in the classical case

Hämmerling, Gutkin, Guhr, EPL 96 (2011) 20007

Transitions

transition operator $\hat{A} = A(\hat{X})$, spectral function

$$S_{\hat{A}}(\omega) \; = \; \sum_{n=1}^{\infty} |\langle 0|A(\hat{X})|n\rangle|^2 \, \delta\left(\omega - \frac{E_n - E_0}{\hbar}\right)$$

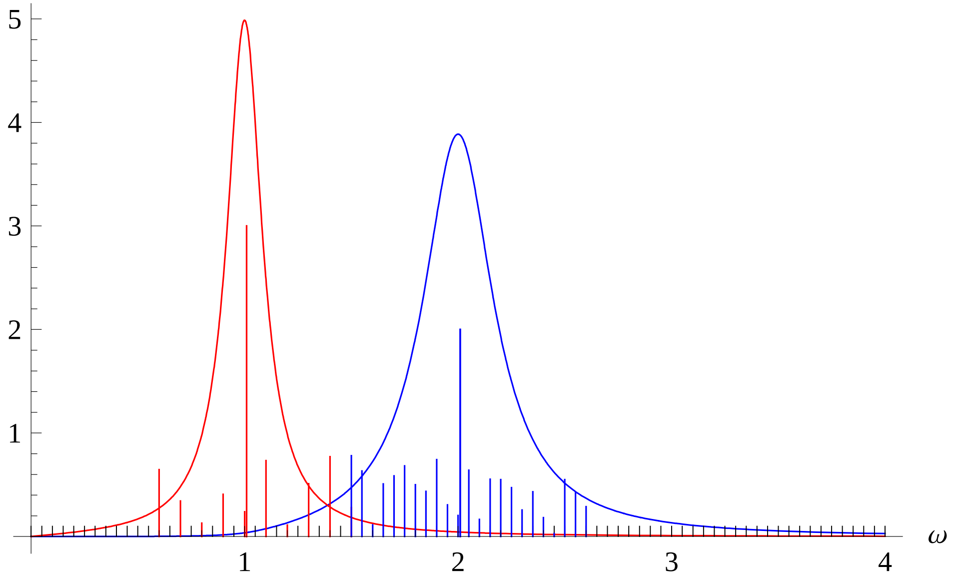
eigenstates $|n\rangle$ and energies E_n of full system

large $|\langle 0|A(\hat{X})|n\rangle|^2 \;\longrightarrow\;$ collective excitation

for $\hat{A} = \hat{X}$, Fourier transform $\mathcal{F}[\mathrm{Im} S_{\hat{X}}](t) \; \sim \; X(t)$
proportional to spread motion of classical collective coordinate

Smoothed Spectral Functions

$|\langle 0|A(\hat{X})|n\rangle|^2$ yields individual peaks, apply smoothing



always Lorentzians!

$\bar{S}_{\hat{X}}(\omega)$ peaks near Ω and $\bar{S}_{\hat{X}^2}(\omega)$ peaks near 2Ω

spreading derived explicitly from the microscopic model

Make Toy Model Chaotic

still N particles in each cloud, extend toy model, add

non–integrable coupling $$H^{(\mathrm{ni})} = \lambda \sum_{i,j=1}^{N} f(x_i^{(1)} - x_j^{(2)})$$

function $f(z)$ positive, even, analytic, otherwise arbitrary
dimensionless strength parameter λ

to leading order: renormalized harmonic Hamiltonian for all $f(z)$, dynamics and spreading as before with renormalized parameters containing quantum corrections

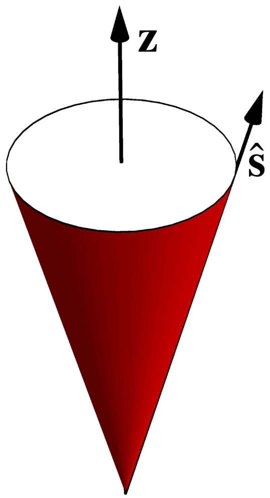
Hämmerling, Gutkin, Guhr, EPL 96 (2011) 20007

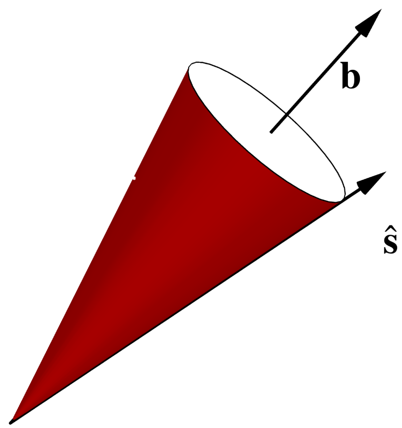
Semiclassics for Spin Chains

Single Kicked Top

Hamiltonian has "Ising" and kicked part with magnetic field $\vec{b}$

$$\hat{H}(t) = \frac{4J\left(\hat{s}^z\right)^2}{(s+1/2)^2} + \frac{2\vec{b}\cdot\hat{\vec{s}}}{(s+1/2)} \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$



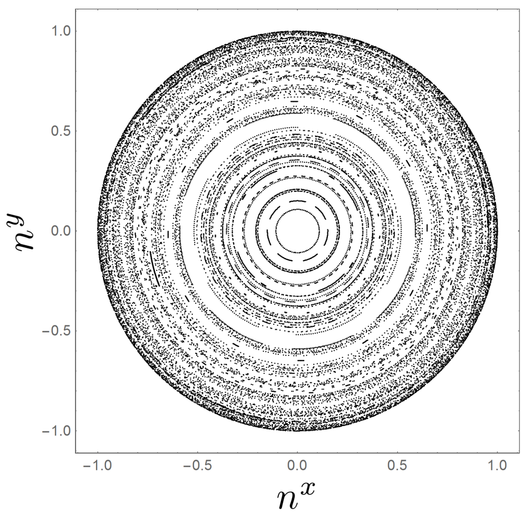


spin quantum number s is semiclassical parameter

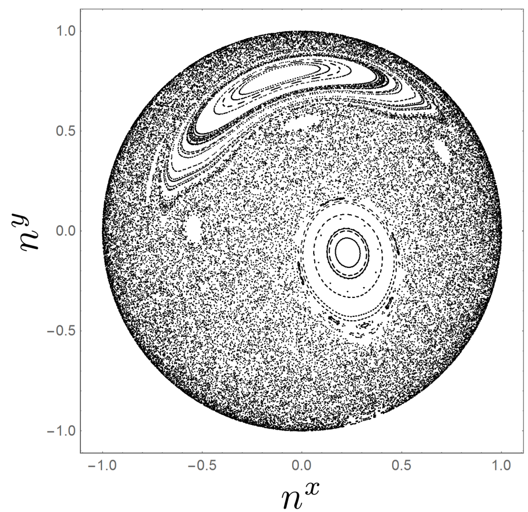
Single Kicked Top — Classical Dynamics

$|\vec{b}|$ fixed, angle defined by $\tan\beta = b^x/b^z$

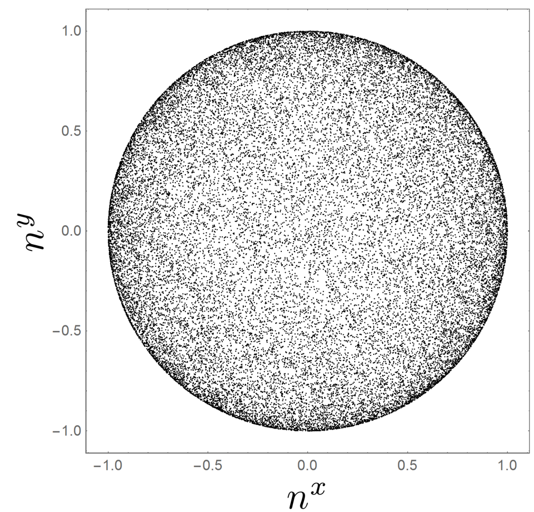
stroboscopic trajectory map on Bloch sphere



$\beta = 0$
regular



$\beta = 0.2$
mixed



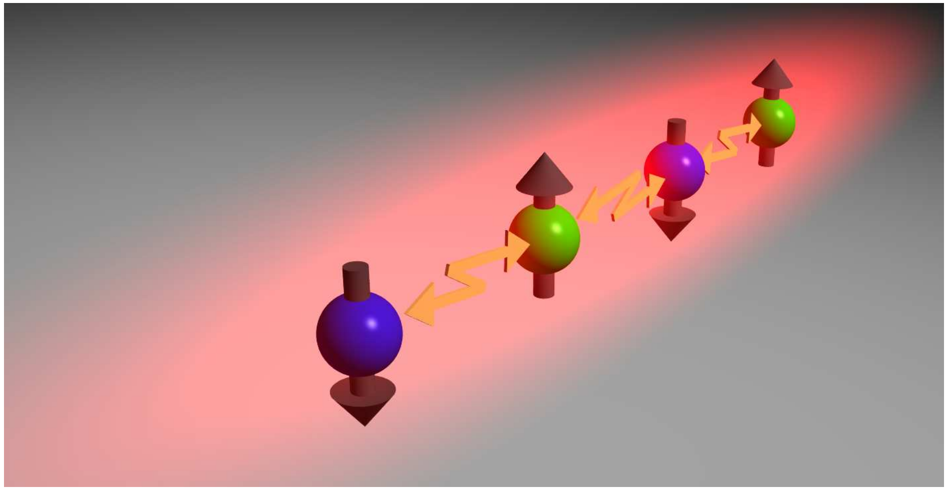
$\beta = \pi/4$
chaotic

experimental realization: Chaudhury et al. Nature (2009)

Many–Body Generalization: Kicked Spin Chain

N coupled spin–s particles with periodic boundary conditions

$$\hat{H}(t) = \sum_{n=1}^{N} \frac{4J\hat{s}^z_{n+1}\hat{s}^z_n}{(s+1/2)^2} + \frac{2}{s+1/2}\sum_{n=1}^{N} \vec{b}\cdot\hat{\vec{s}}_n \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$



periodic boundary conditions:

$$\hat{\vec{s}}_{N+1} = \hat{\vec{s}}_1$$

experiments for $s = 1/2$ in groups of
Greiner (Harvard), Jochim (Heidelberg), Bloch (Munich)

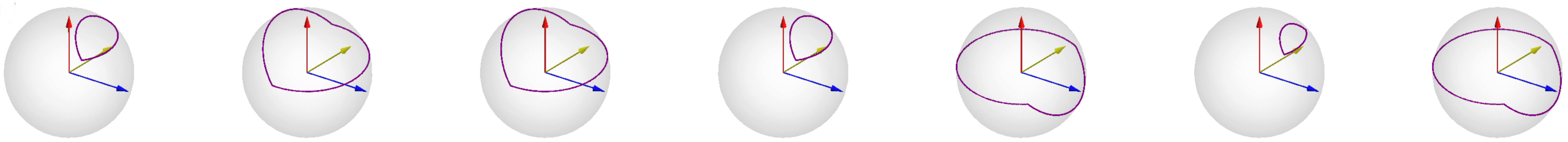
Classical Dynamics of Kicked Spin Chain

unit vector $\vec{s}_n(t)$ on the Bloch sphere for spin n

rotations $\vec{s}_n(t+1) = R_{\vec{z}}(4J\chi_n) R_{\vec{b}}(2|\vec{b}|)\, \vec{s}_n(t)$

angle $\chi_n = s^z_{n-1} + s^z_{n+1}$

example: periodic orbits for $N = 7$ spins, $T = 1$ kick



Semiclassics for Large Spin Quantum Number

semiclassical limit: $s \;=\; 1/\hbar_{\mathrm{eff}} \;\to\; \infty$

many–body periodic orbits: trace formula for time evolution

$$\mathrm{Tr}\,\hat{U}^T \;=\; \int da \langle a|\hat{U}^T|a\rangle \;\sim\; \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma}$$

with periodic orbits $\gamma(T)$, stability prefactor A_γ and action S_γ of many–body system

Fourier transform yields spectrum of classical actions

$$\rho(S) \;\sim\; \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS}\mathrm{Tr}\,\hat{U}^T \;\sim\; \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

Serious Problem

and Miraculous Solution

Explosion of Dimensions and Duality Relation

huge dimensions $\dim \hat{U} \; = \; (2s+1)^N \times (2s+1)^N$

example: $s = 10$, $N = 20$ yields $(2s+1)^N = 2.8 \cdot 10^{26}$

MIRACLE: duality of propagations in time and particle number

time evolution $\hat{U}$, particle number evolution $\hat{W}$

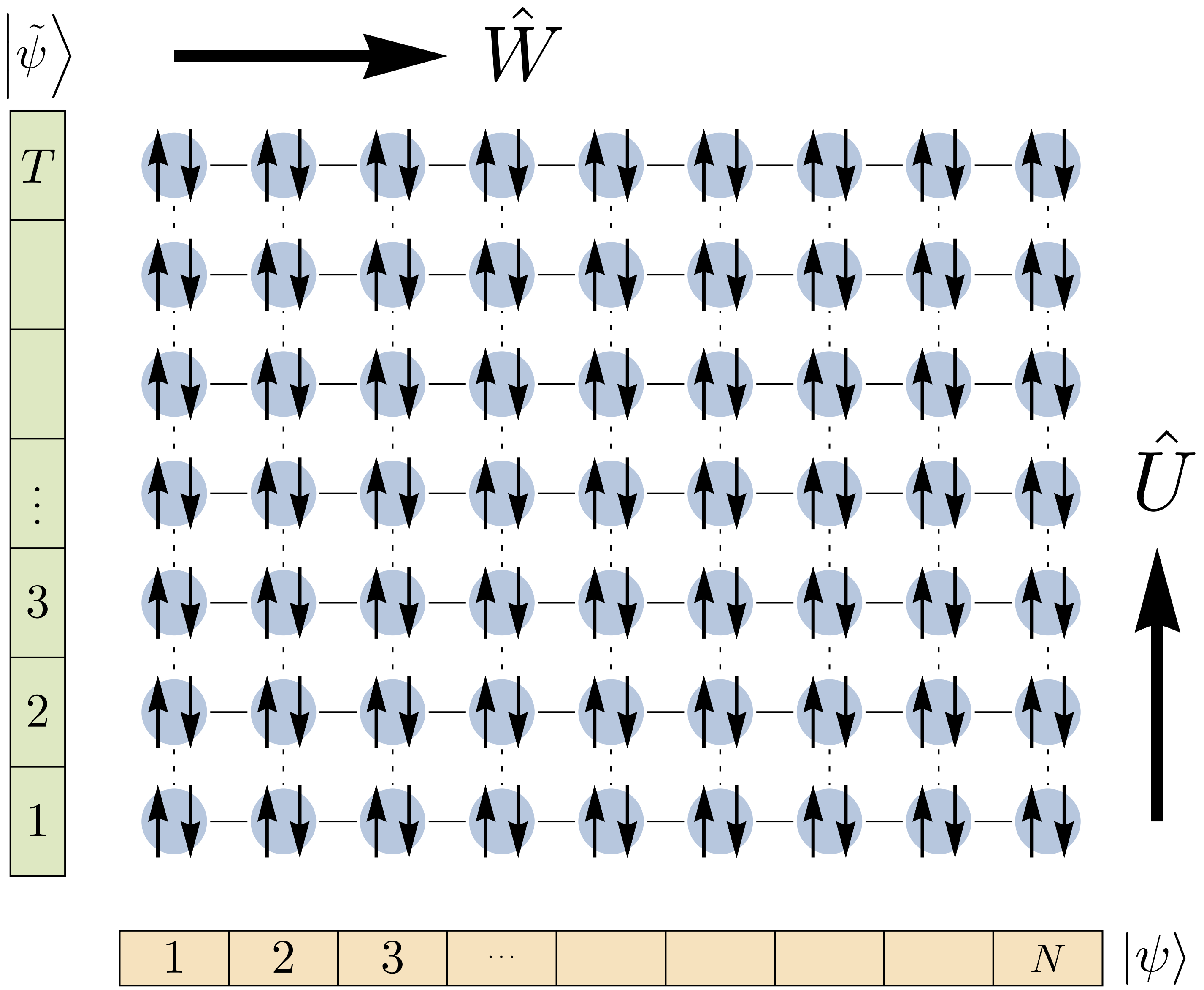
duality relation $\mathrm{Tr}\,\hat{U}^T \; = \; \mathrm{Tr}\,\hat{W}^N$

dimension $\dim \hat{W} \; = \; (2s+1)^T \times (2s+1)^T$

we can calculate short time behavior for many quantum spins !

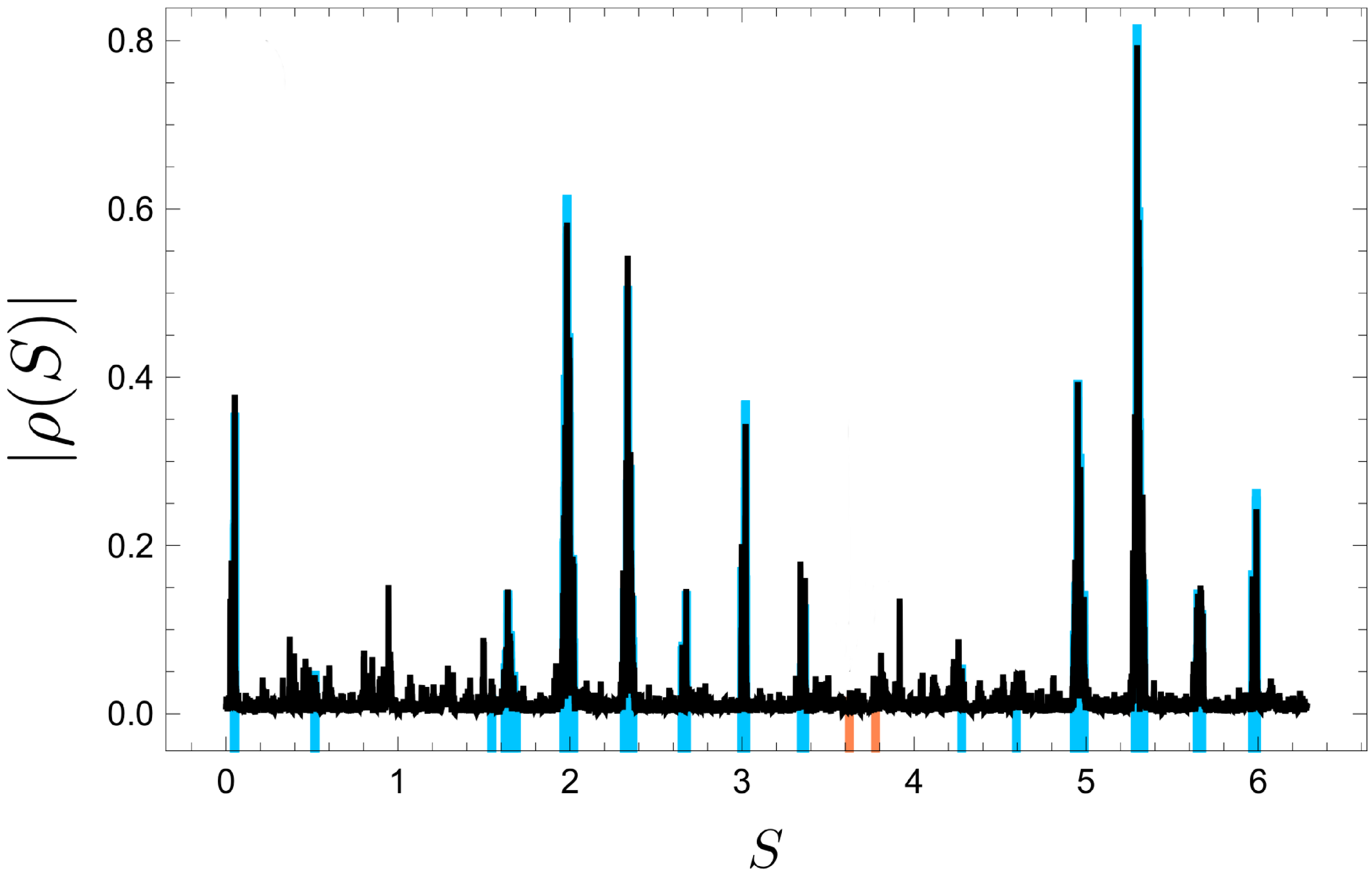
Gutkin, Osipov, Nonlinearity (2016); Akila, Waltner, Gutkin, Guhr, J. Phys. A (2016)

Cartoon–type–of Visualization of Duality



Classical Action Spectrum for One Kick, $T = 1$

example: $N = 19$ spins, $J = 0.7$, $b^x = b^z = 0.9$, $s_{\text{cut}} = 4650$

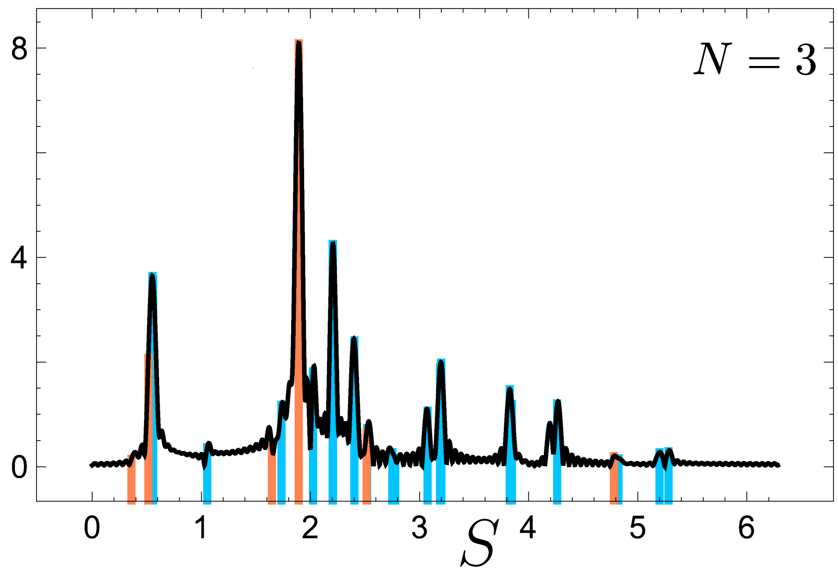


Dominance of Collectivity

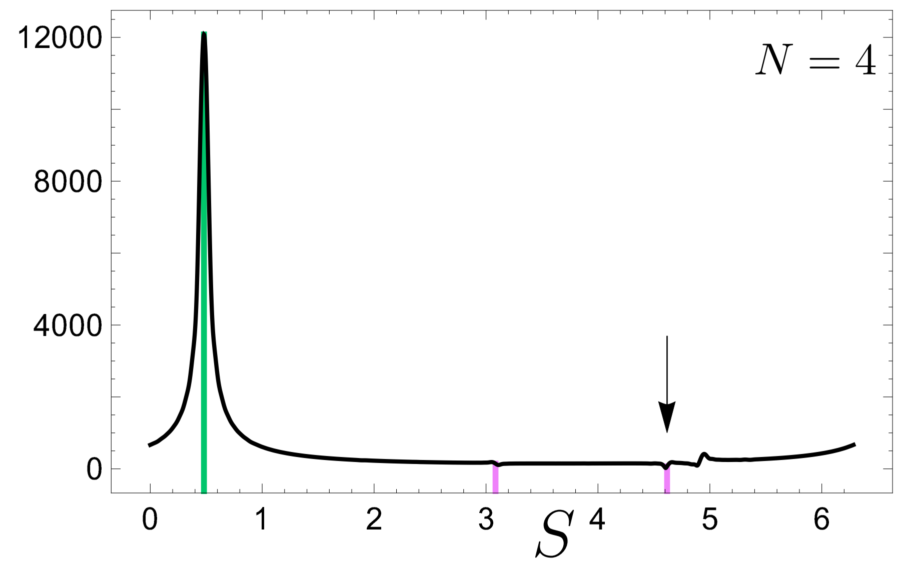
Emergence of Collectivity for Two Kicks, $T = 2$

parameters $J = 0.7$, $b^x = b^z = 0.9$, $s_{\text{cut}} = 114$

$N = 3$ $N = 4$



incoherent motion

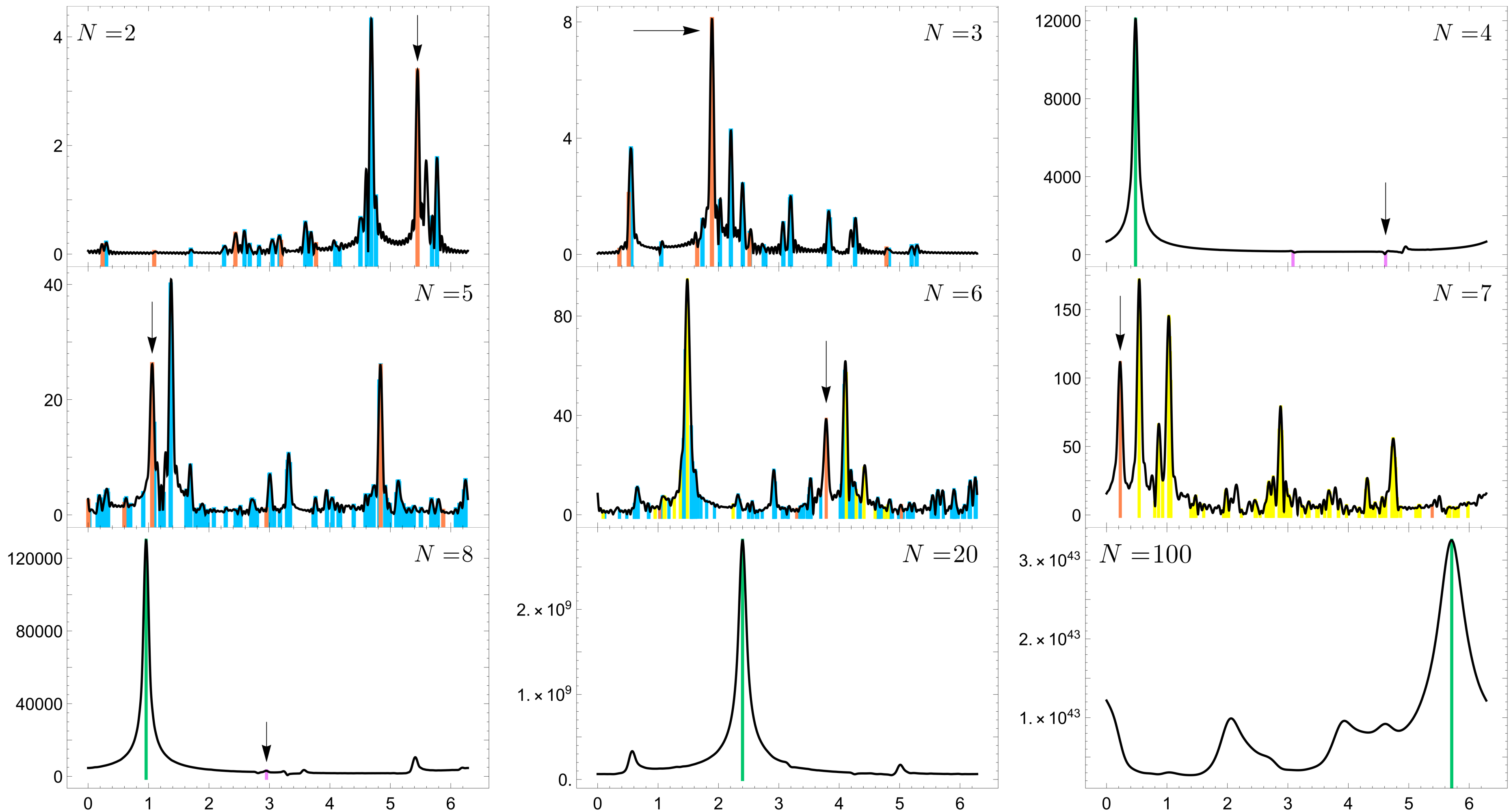


collective motion

Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017)

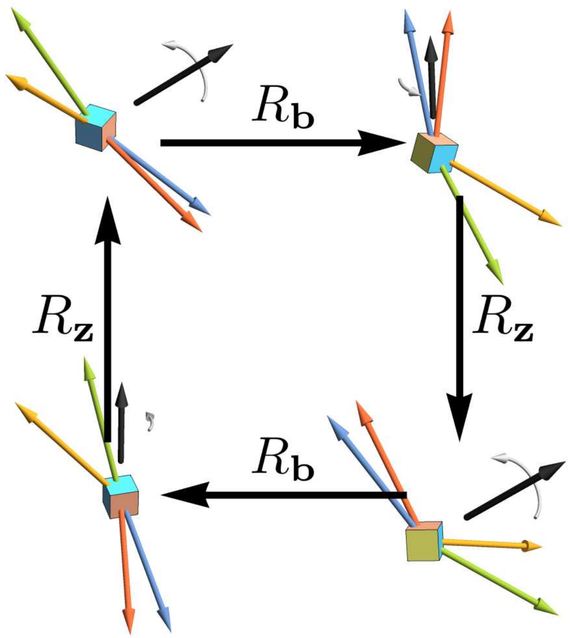
A Multiple–of–Four Collectivity

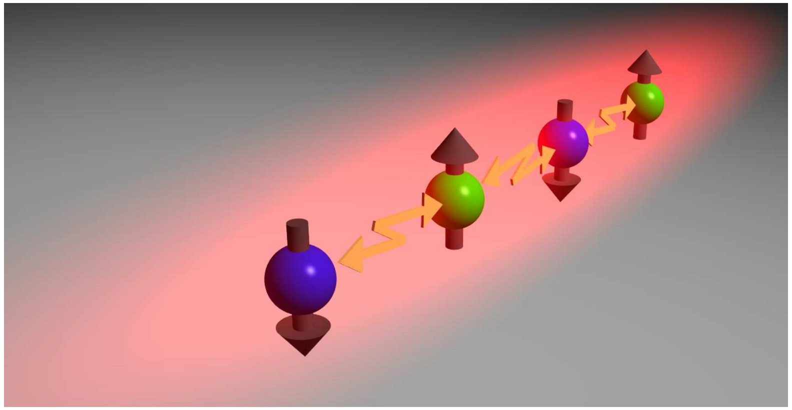
observation generalizes whenever $N = 4k$ with integer k



Semiclassical Explanation of Collective Motion

four–dimensional manifold of non–isolated periodic orbits with equal actions: four spins perform solid body rotation





blue spins influenced by green ones and vice versa

Akila, Waltner, Gutkin, Braun, Guhr, Phys. Rev. Lett. (2017)

Summary and Conclusions

- chaos in single–particle systems largely understood, including reason for random matrix statistics
- these insights do not carry over to many–body systems, where collective dynamics occurs
- big goal: emergence of collective motion from microscopic models, semiclassics and periodic orbit structure thereof
- exact derivation of spreading from microscopic toy model with full control over particle dynamics
- semiclassics for kicked spin chains using duality relation
- treat incoherent and collective motion on equal footing
- dominance of collectivity counteracts universality due to incoherent single–particle motion