Lattice Models with Discrete Randomness

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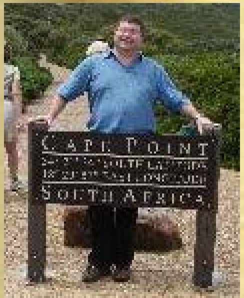
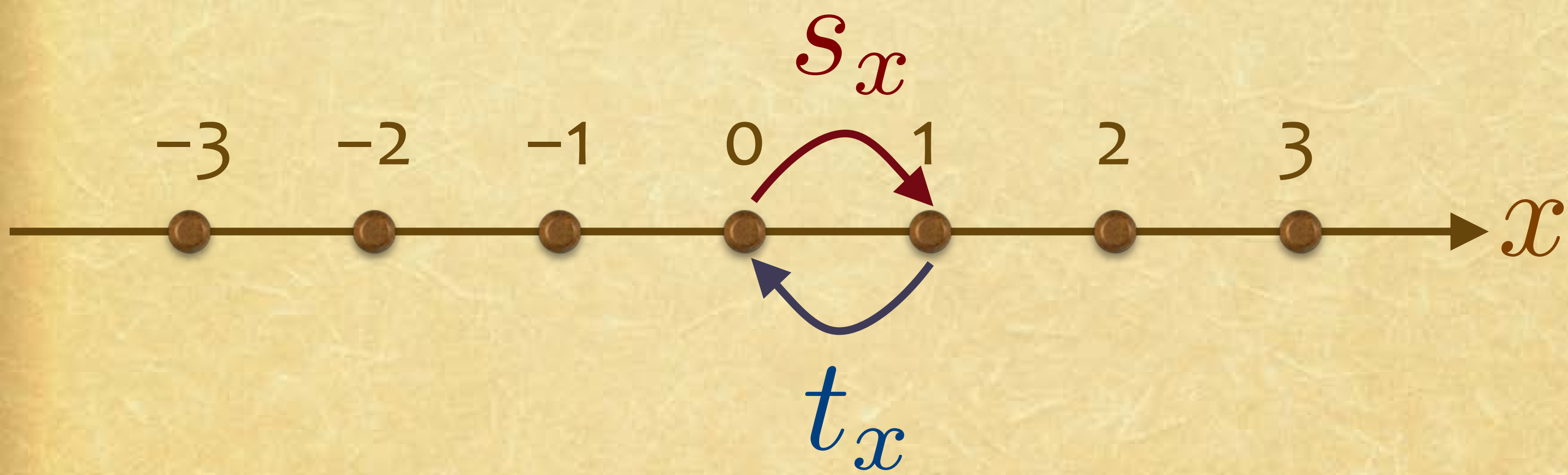


Feinberg-Zee Model

J. Feinberg and A. Zee, PRE **59** (1999) 6433



$$H = \sum_{x=-\infty}^{\infty} \left(s_x |x+1\rangle\langle x| + t_x |x\rangle\langle x+1| \right)$$

random-sign hopping

$$s_x, t_x = \pm 1$$

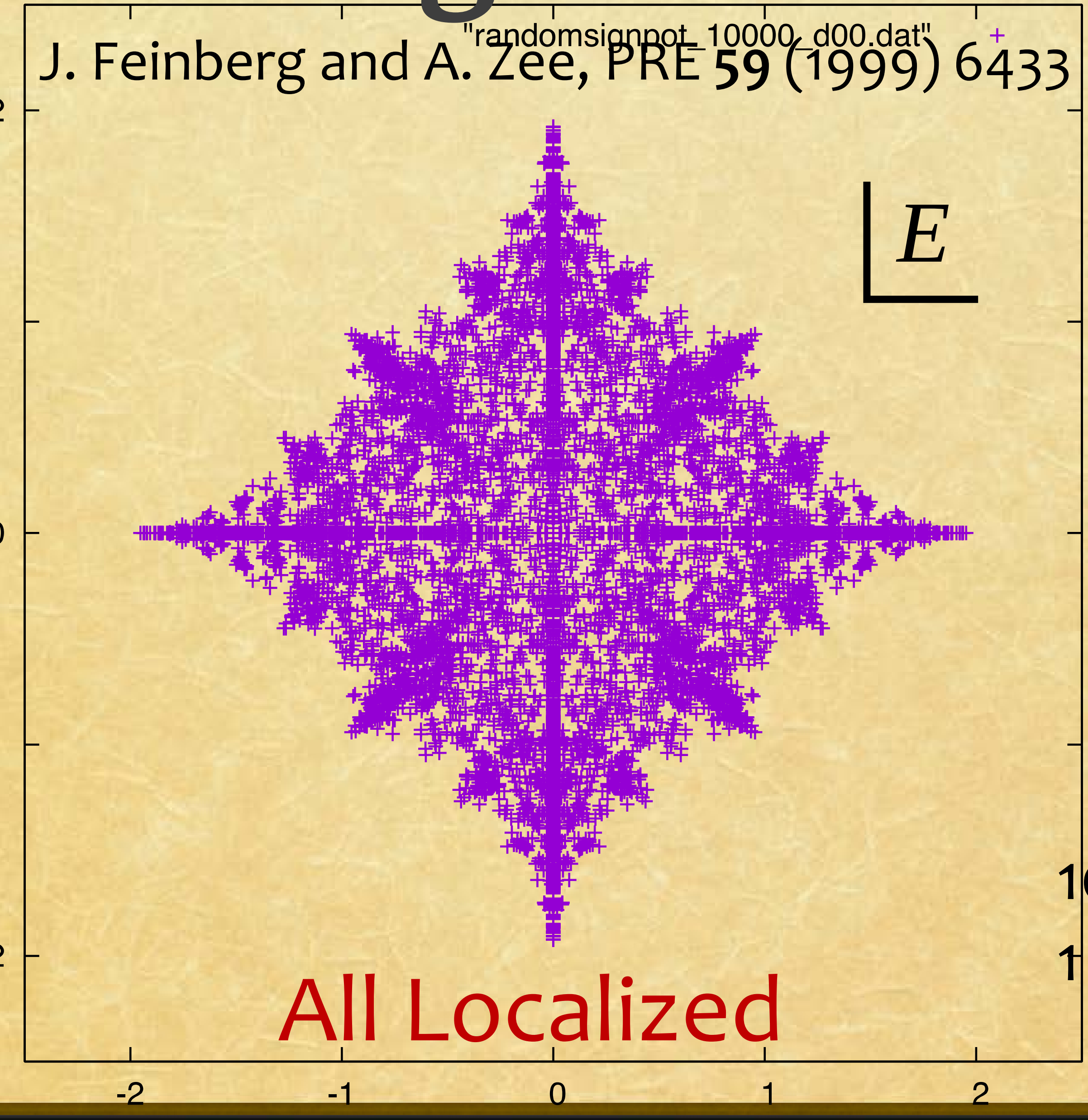
Feinberg-Zee Model

J. Feinberg and A. Zee, PRE **59** (1999) 6433

$$H = \begin{pmatrix} \ddots & \ddots & & & & +1 \\ \ddots & 0 & +1 & & & \\ & -1 & 0 & -1 & & \\ & & -1 & 0 & -1 & \\ & & & +1 & 0 & \ddots \\ -1 & & & & \ddots & \ddots \end{pmatrix}$$

Feinberg-Zee Model

J. Feinberg and A. Zee, PRE **59** (1999) 6433



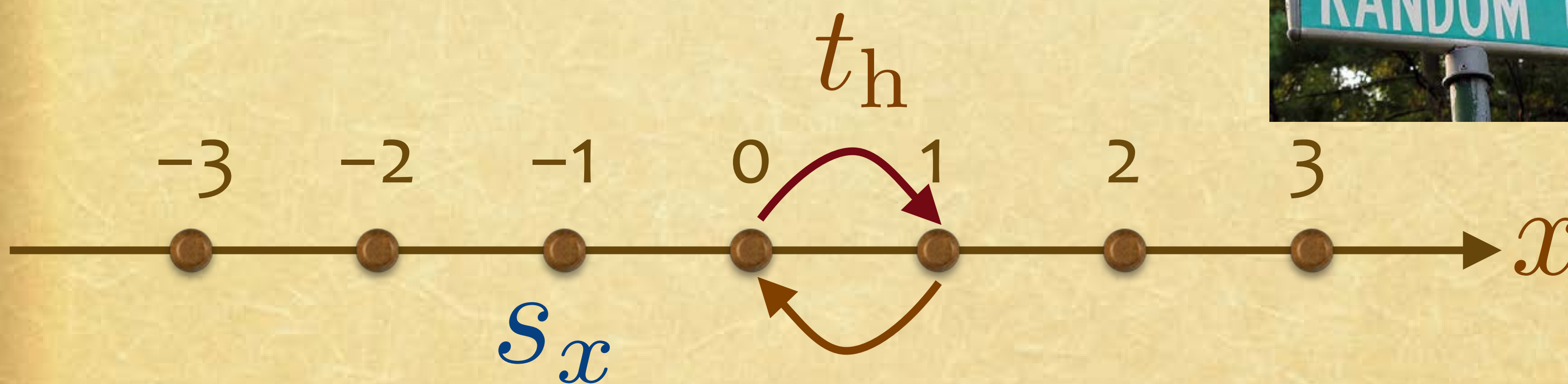
10000 sites

1 sample

All Localized

o sites
nple

Random-Sign Potential



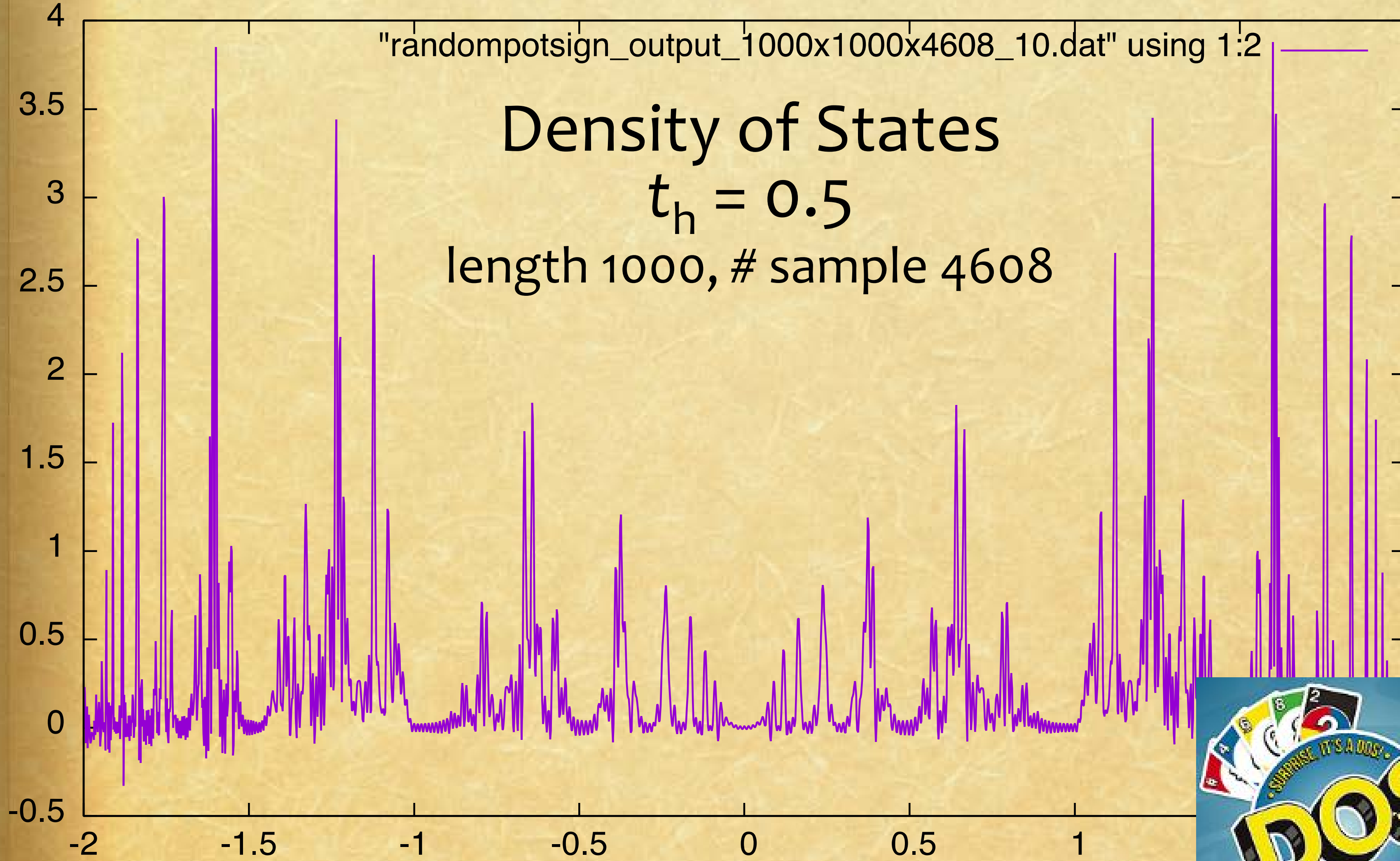
$$H = -t_{\mathrm{h}} \sum_{x=-\infty}^{\infty} \left(|x+1\rangle\langle x| + |x\rangle\langle x+1| \right)$$

$$= \sum_{x=-\infty}^{\infty} s_x |x\rangle\langle x| \qquad s_x = \pm 1$$

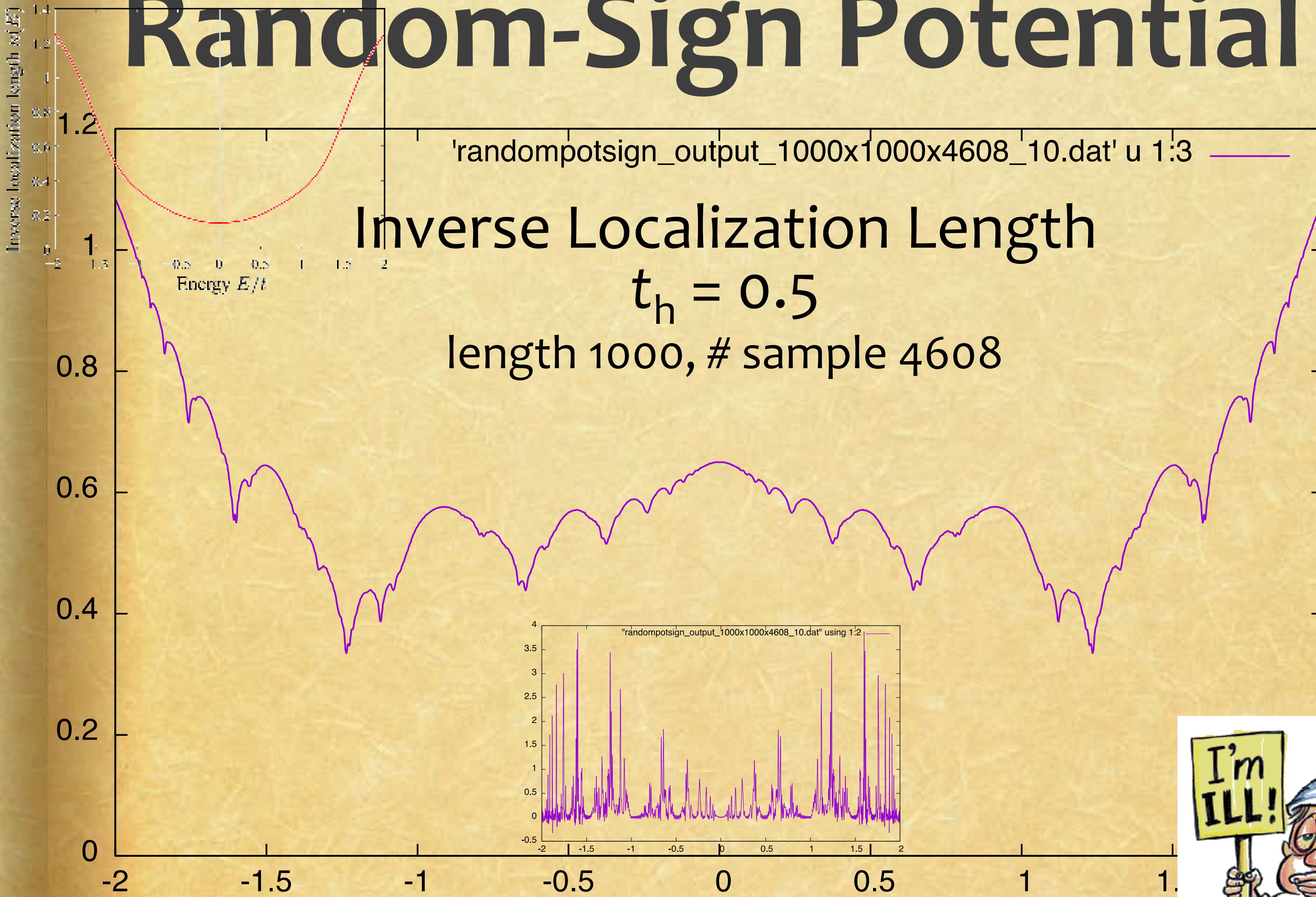
Random-Sign Potential

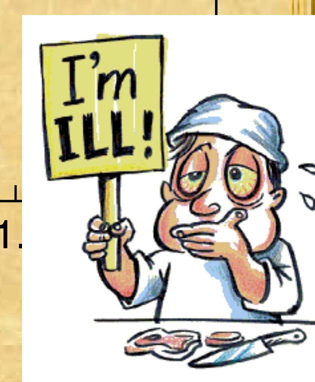
$$H = \begin{pmatrix} \ddots & \ddots & & & & -t_{\mathrm{h}} \\ \ddots & +1 & -t_{\mathrm{h}} & & & \\ & -t_{\mathrm{h}} & -1 & -t_{\mathrm{h}} & & \\ & & -t_{\mathrm{h}} & -1 & -t_{\mathrm{h}} & \\ & & & -t_{\mathrm{h}} & +1 & \ddots \\ -t_{\mathrm{h}} & & & & \ddots & \ddots \end{pmatrix}$$

Random-Sign Potential

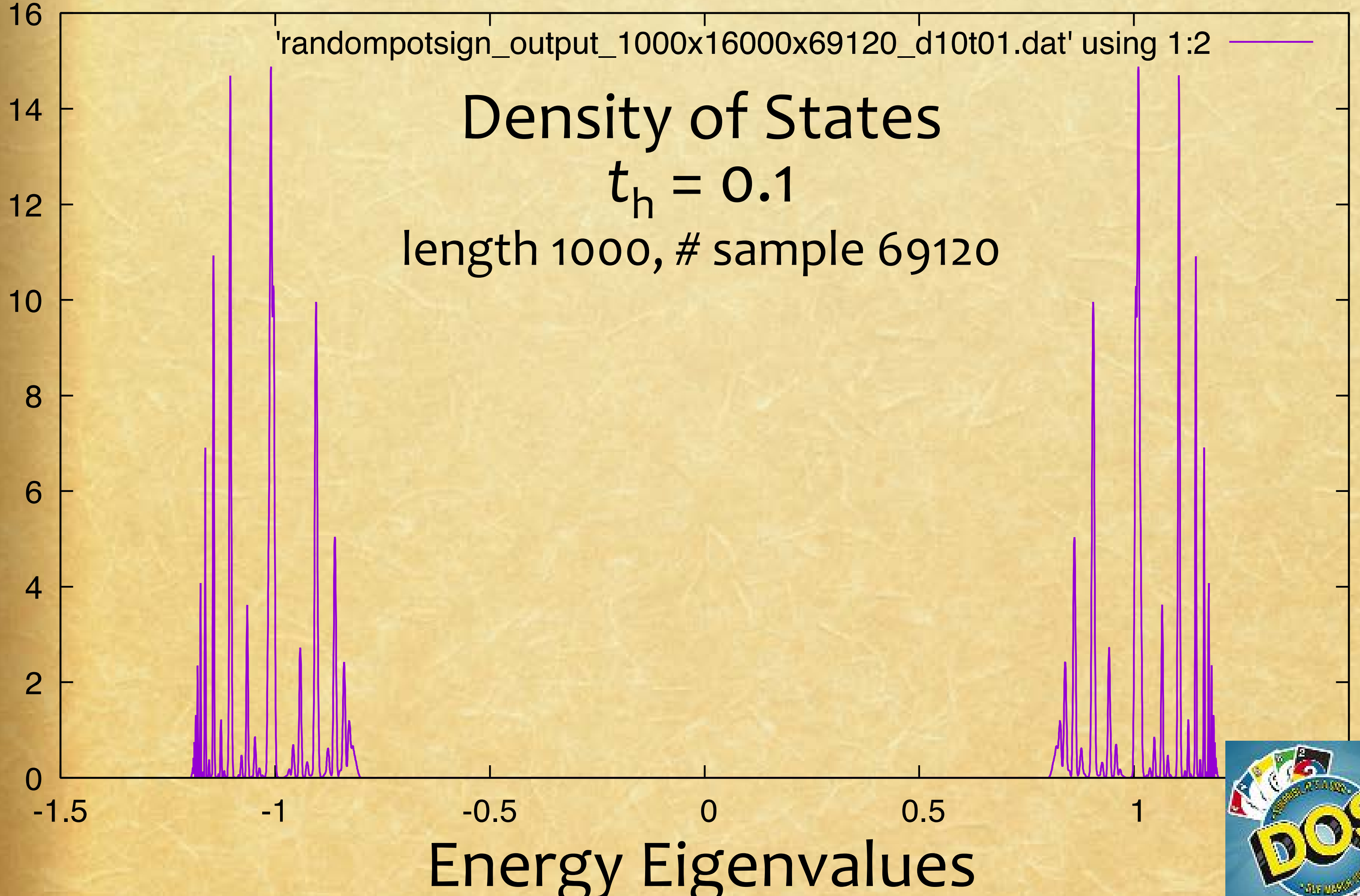


Random-Sign Potential

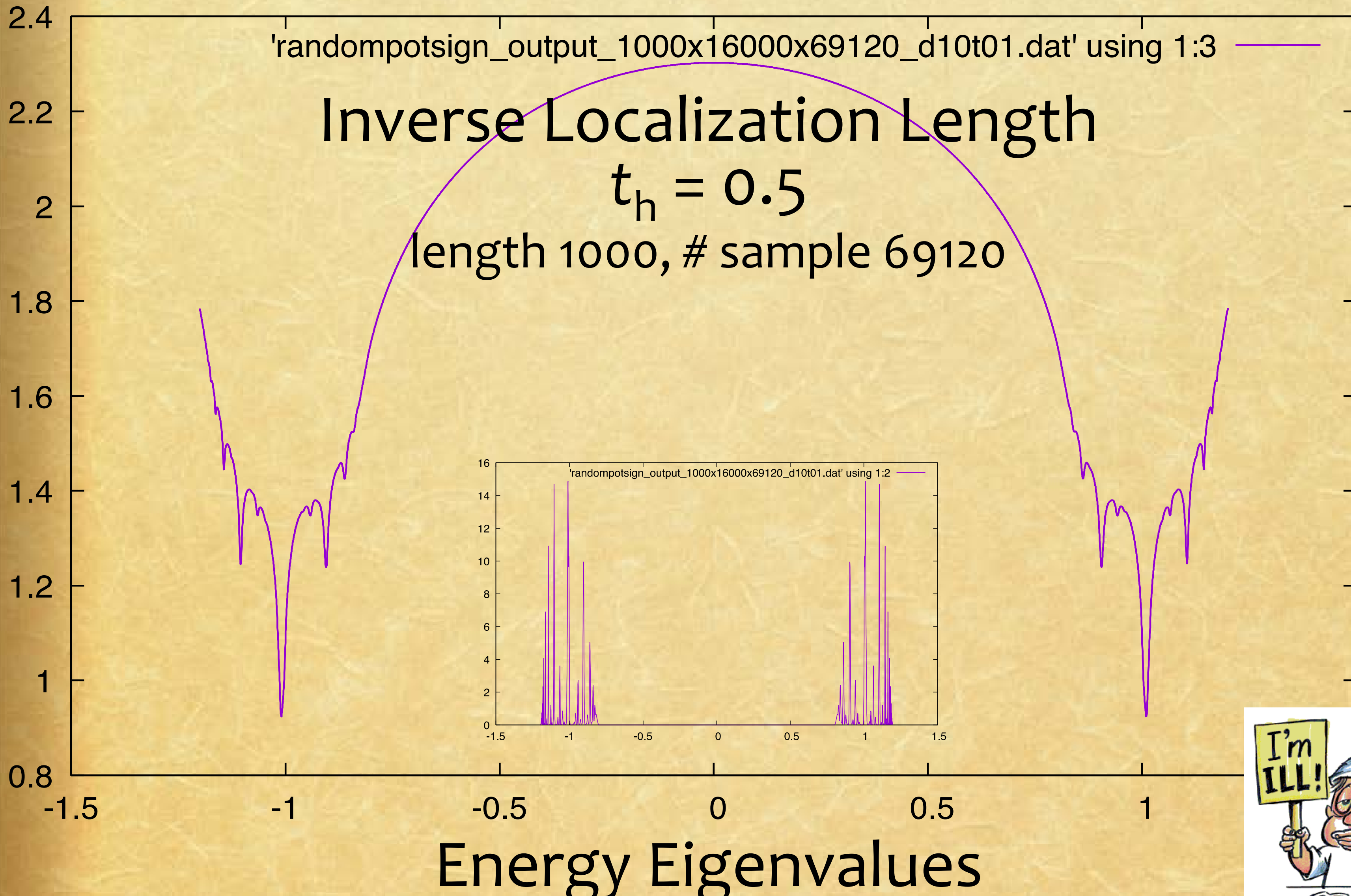




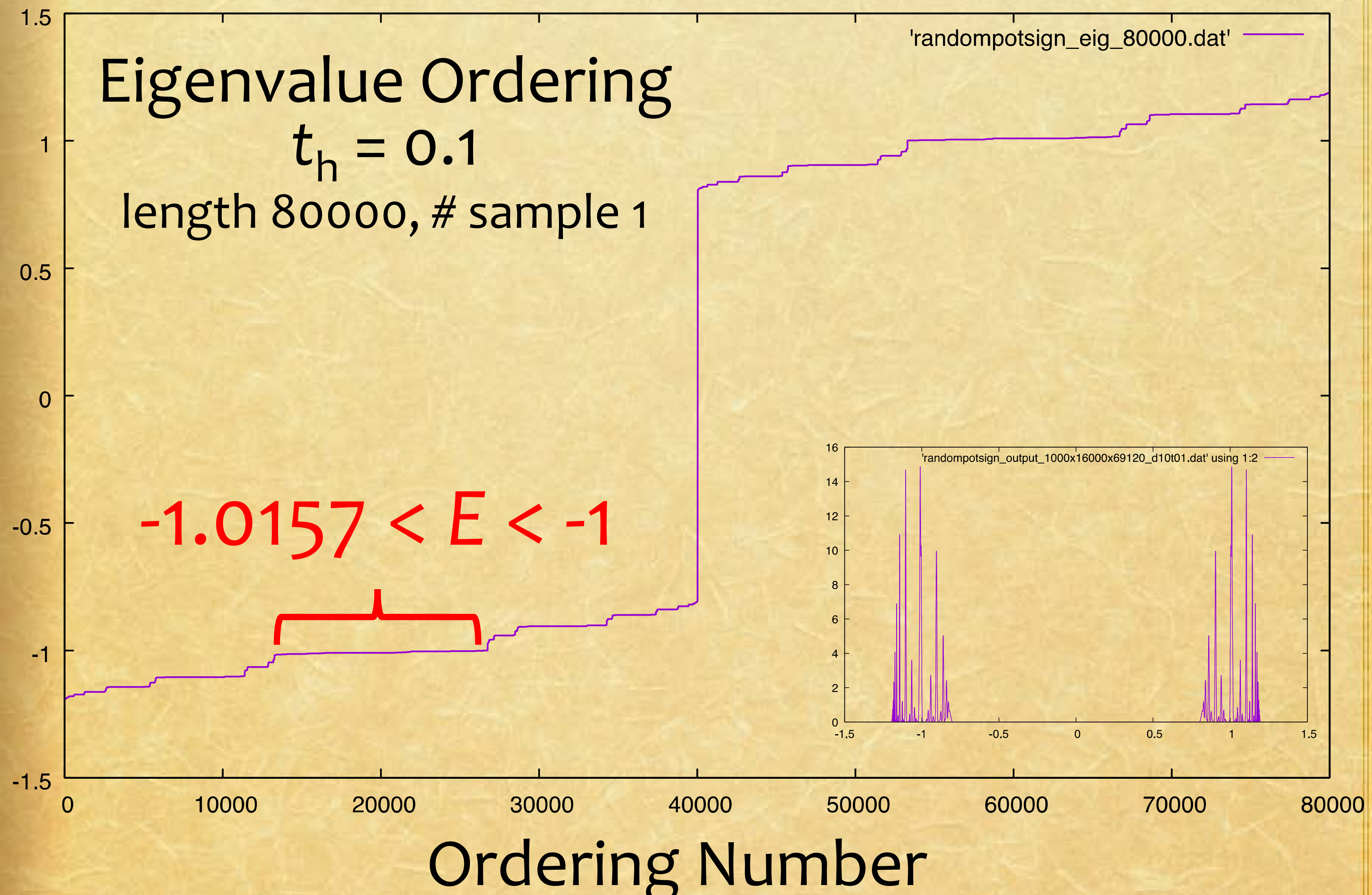
Random-Sign Potential



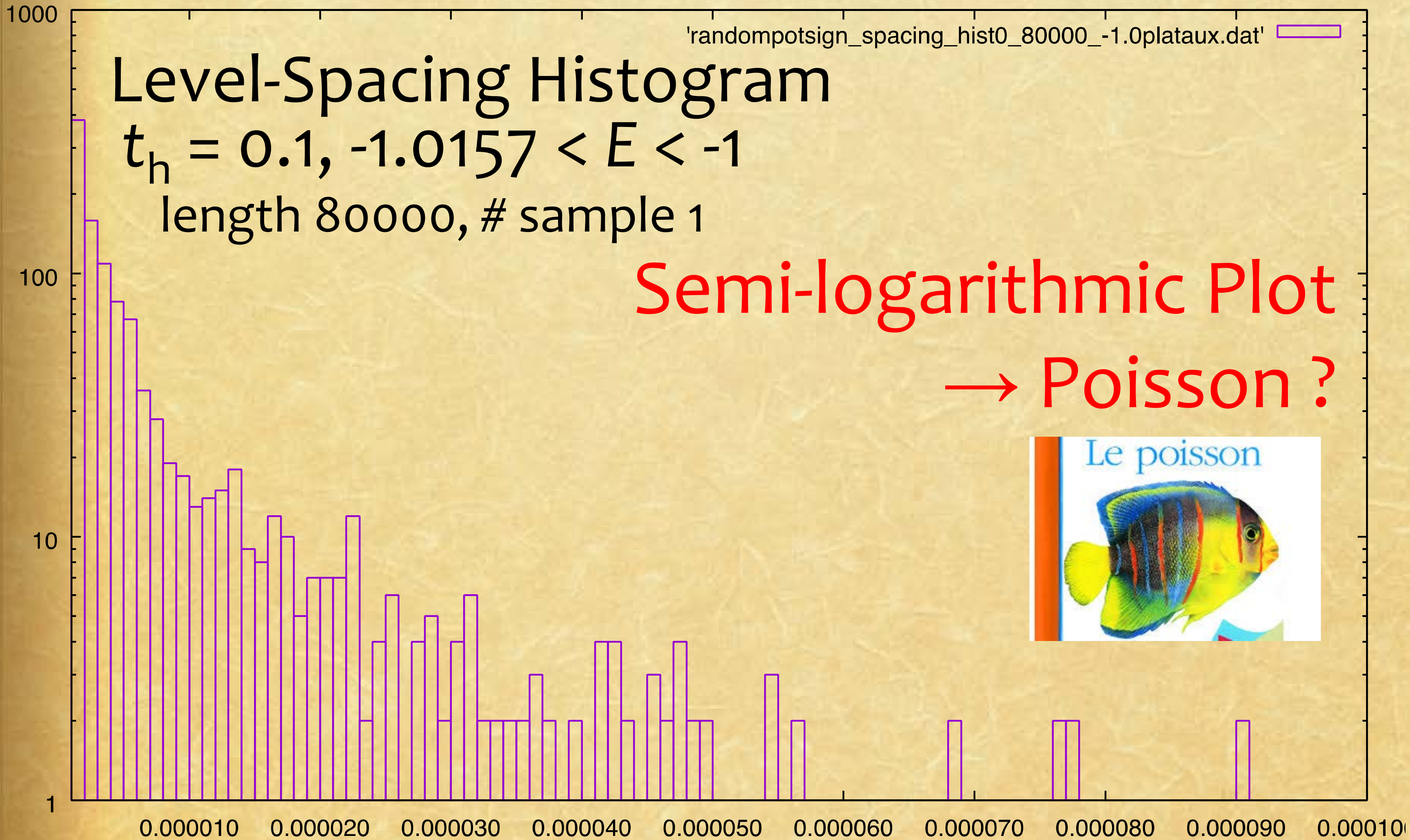
Random-Sign Potential



Random-Sign Potential

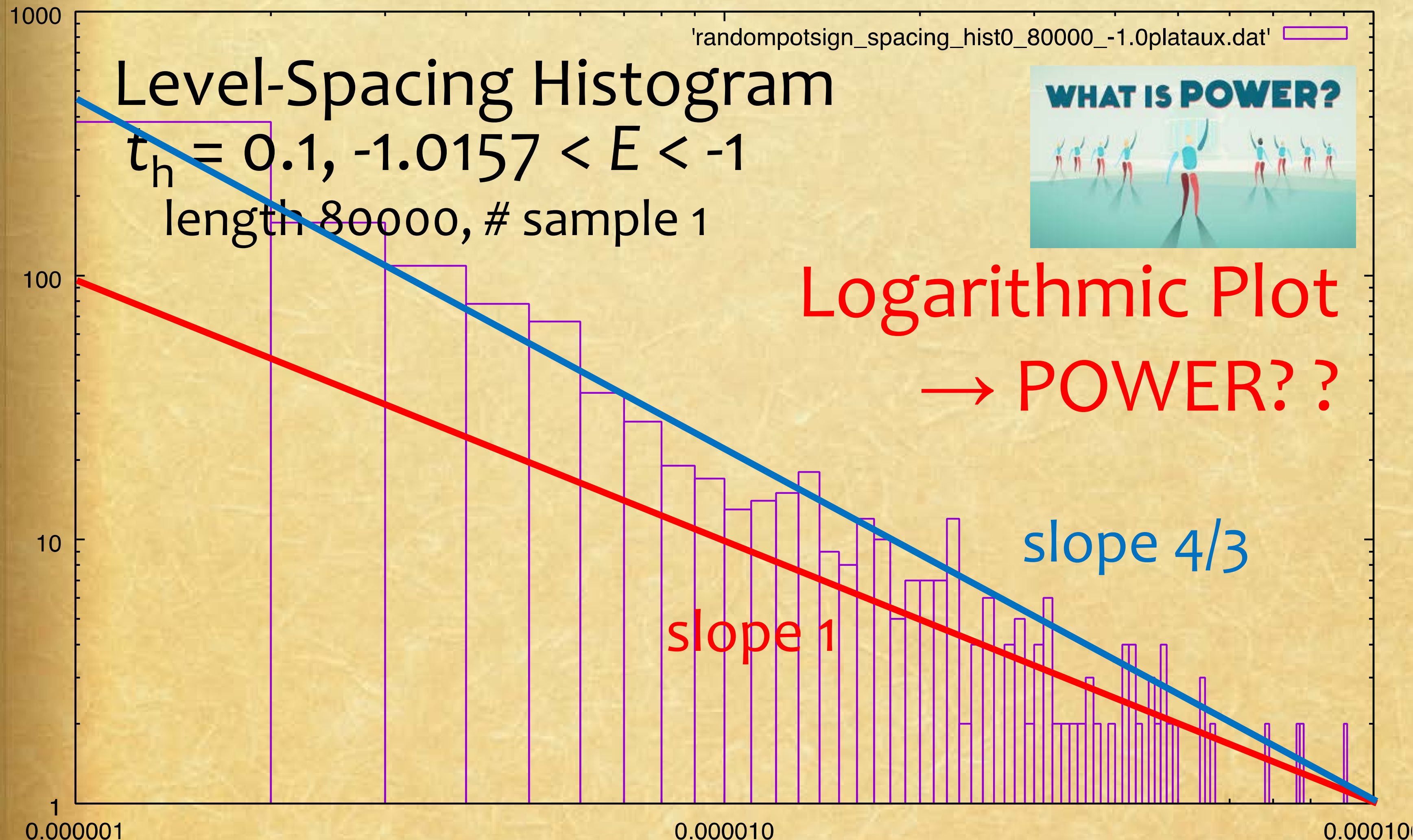


Random-Sign Potential



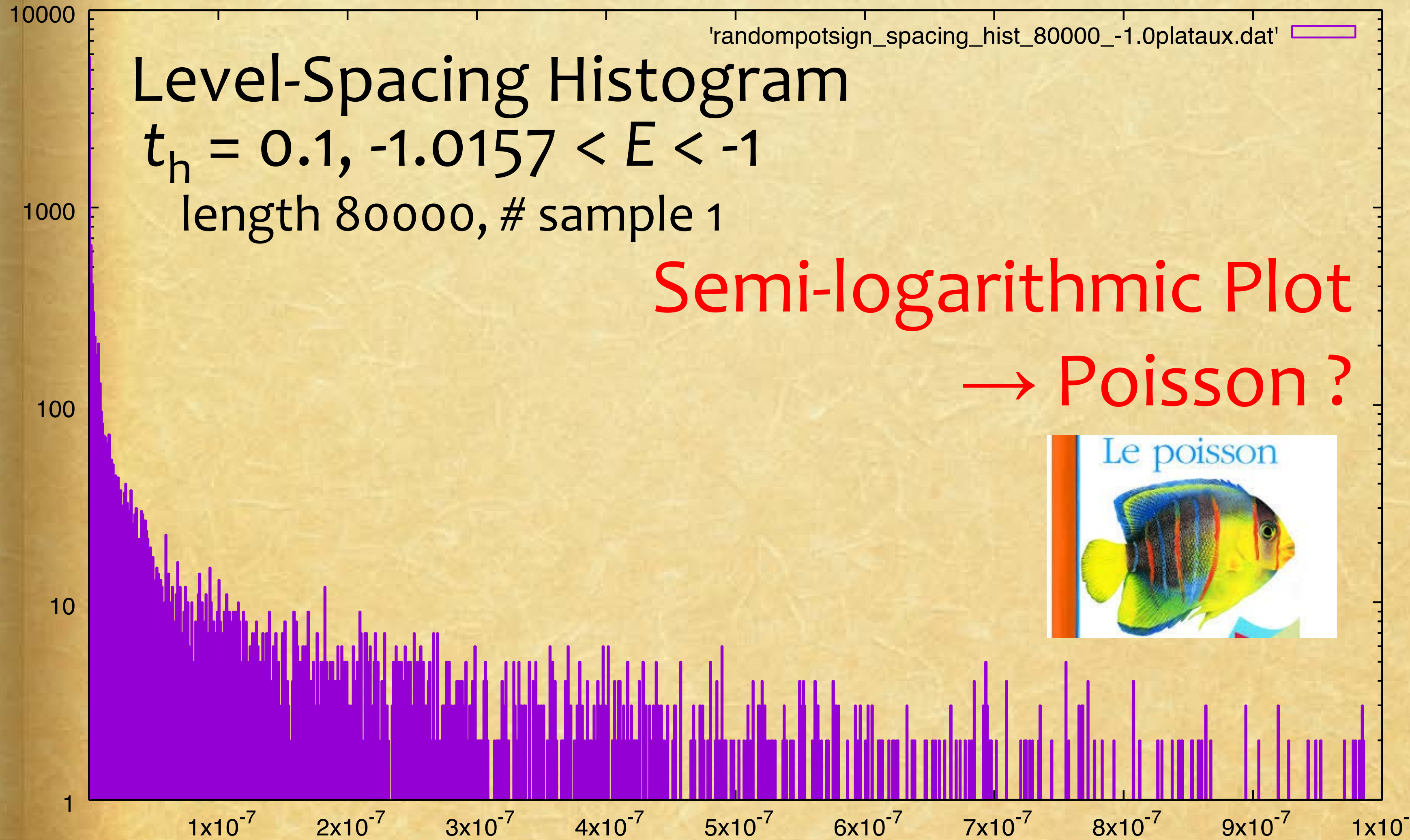
Level Spacing (Bin Width $10^{-6} \sim t_h/L$)

Random-Sign Potential



Level Spacing (Bin Width $10^{-6} \sim t_h/L$)

Random-Sign Potential



Level-Spacing Histogram

t_h = 0.1, -1.0157 < E < -1

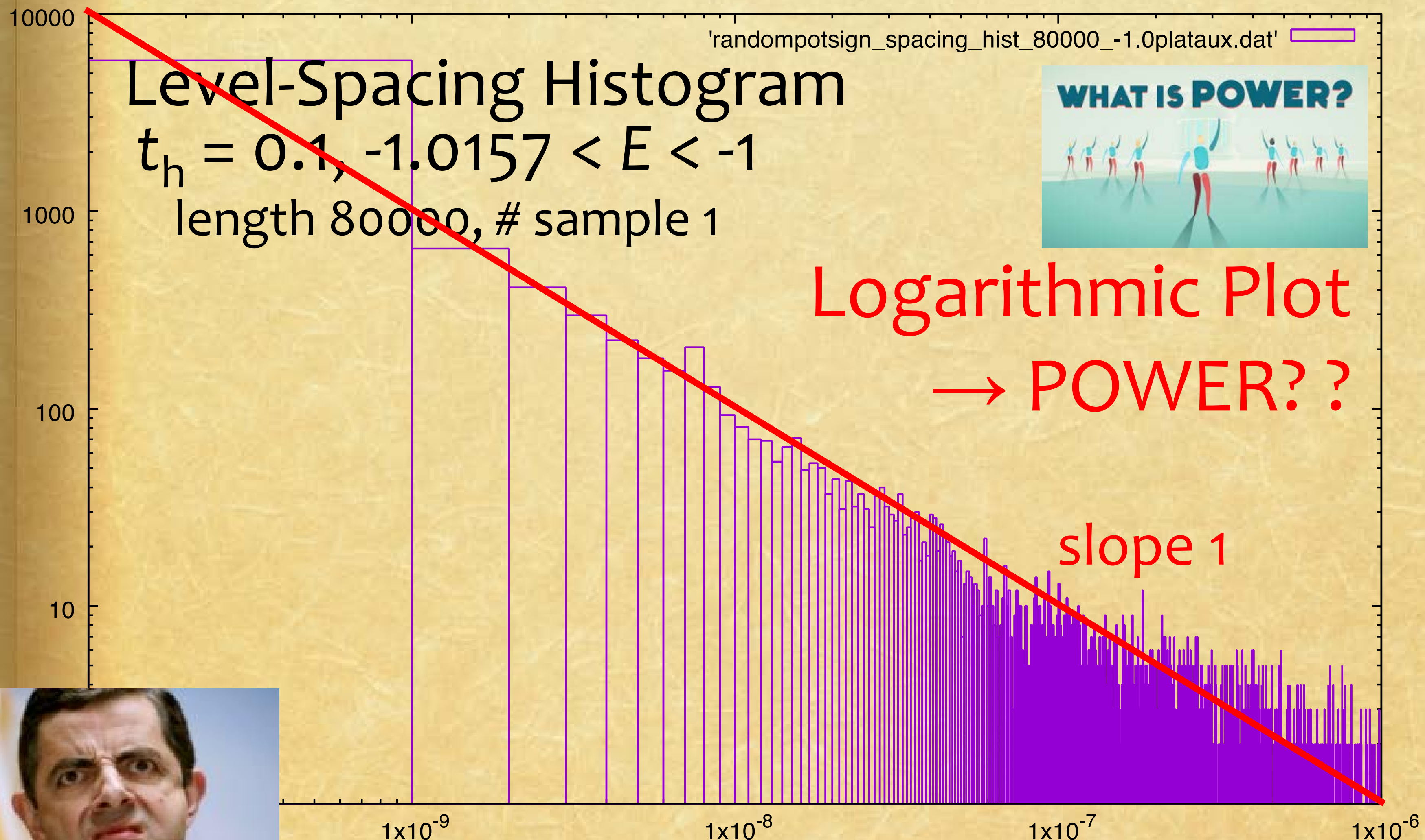
length 80000, # sample 1

Semi-logarithmic Plot

→ Poisson ?

Level Spacing (Bin Width $10^{-9} \ll t_h/L$)

Random-Sign Potential



Level-Spacing Histogram

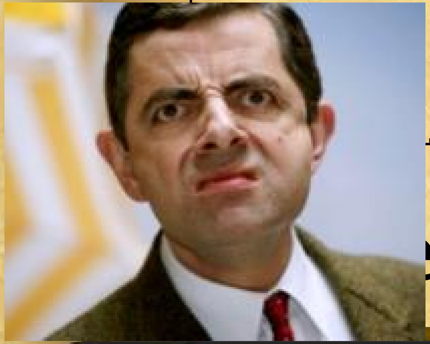
$t_h = 0.1$, $-1.0157 < E < -1$

length 80000, # sample 1

Logarithmic Plot

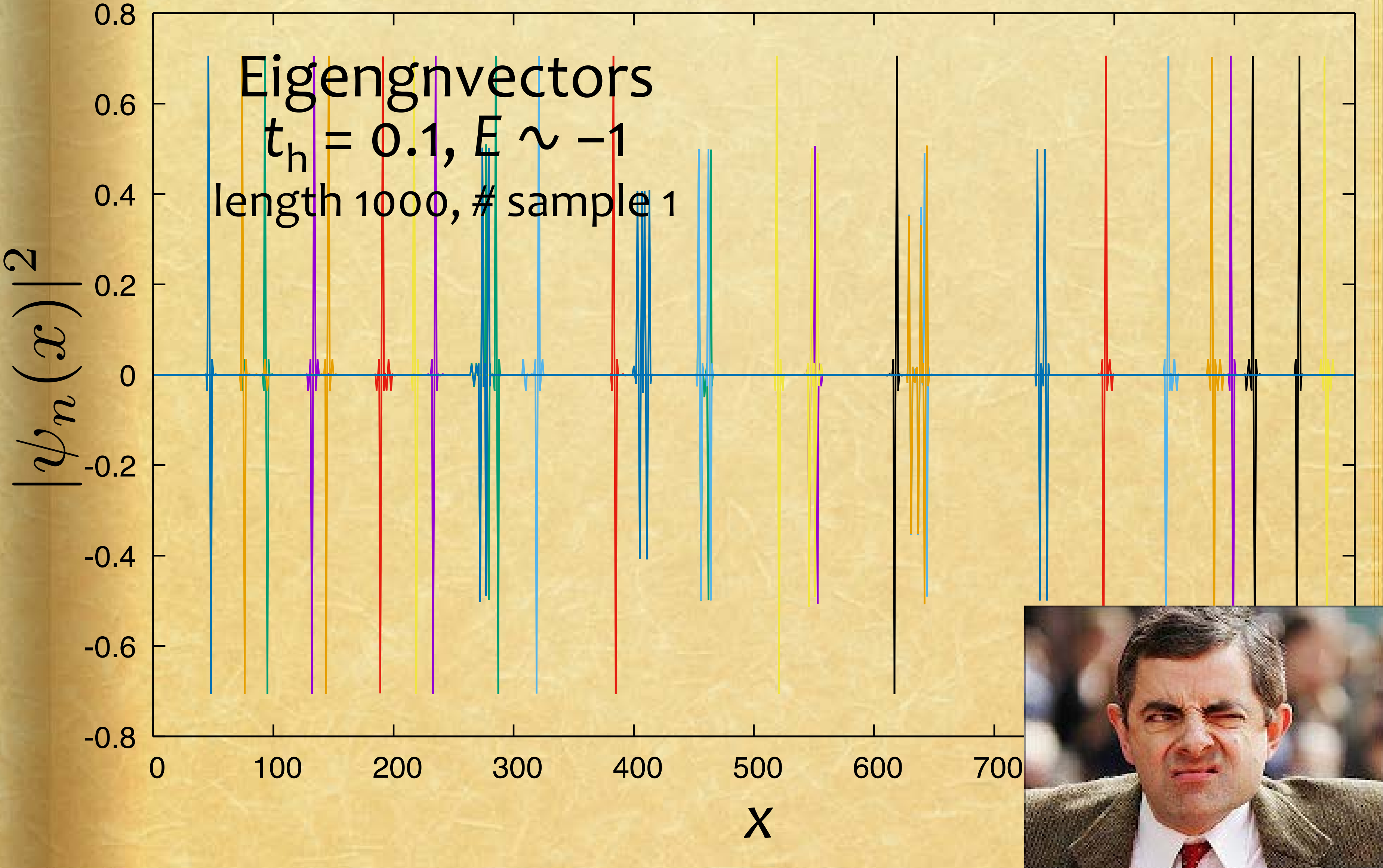
→ POWER? ?

slope 1



evel Spacing (Bin Width $10^{-9} \ll t_h/L$)

Random-Sign Potential



Chebyshev Polynomial Expansion of the density of states

R.N. Silver and H. Röder (1994)

N×*N* Hermitian matrix: *H*

$$\rho(E) = \frac{1}{N}\sum_{\mu=1}^{N}\delta(E - E_\mu)$$

$$= \frac{1}{\sqrt{1-E^2}}\sum_{n=0}^{\infty} c_n T_n(E)$$

$T_n(E)$: Chebyshev polynomial

Chebyshev Polynomial Expansion of the density of states

R.N. Silver and H. Röder (1994)

$$\int_{-1}^{1} T_n(E) T_m(E) \frac{dE}{\sqrt{1-E^2}} = \frac{\pi}{2}\delta_{nm}$$

$$\rho(E) = \frac{1}{\sqrt{1-E^2}} \sum_{n=0}^{\infty} c_n T_n(E)$$

$$c_n = \frac{2}{\pi}\int_{-1}^{1} T_n(E)\rho(E)\,dE$$

Chebyshev Polynomial Expansion of the density of states

R.N. Silver and H. Röder (1994)

$$c_n = \frac{2}{\pi}\int_{-1}^{1} T_n(E)\rho(E)dE$$

$$= \frac{2}{N\pi}\sum_{\mu=1}^{N} T_n(E_\mu) = \frac{2}{N\pi}\,\mathrm{Tr}\,T_n(H)$$

Recursive Relation

$$T_{n+1}(H) = 2HT_n(H) - T_{n-1}(H)$$

Chebyshev Polynomial Expansion of the density of states

R.N. Silver and H. Röder (1994)

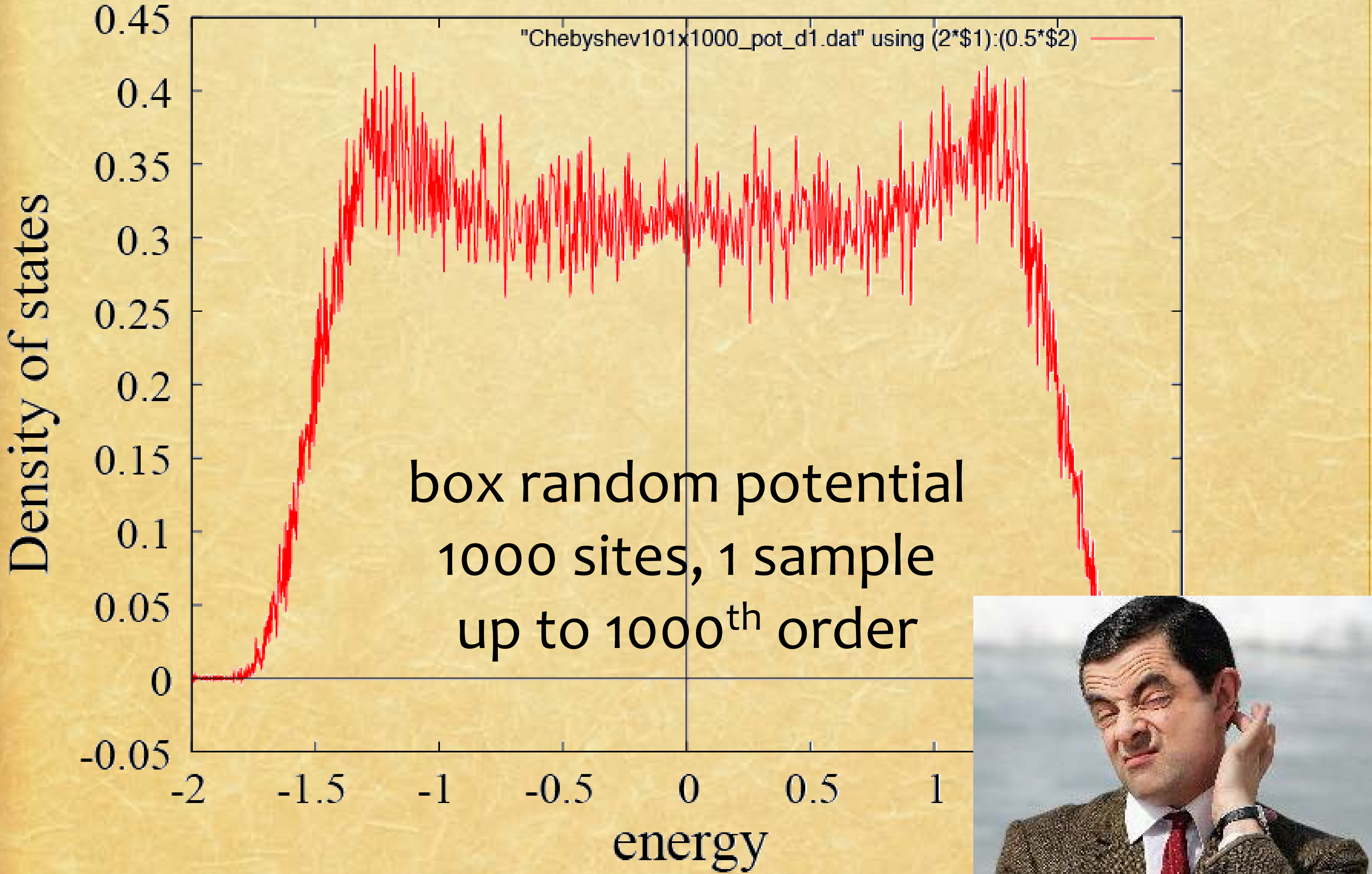
(i) $T_{n+1}(H) = 2HT_n(H) - T_{n-1}(H)$

(ii) $c_n = \frac{2}{N\pi} \mathrm{Tr}\, T_n(H)$

cutoff

(iii) $\rho(E) = \frac{1}{\sqrt{1-E^2}} \sum_{n=0}^{\infty} c_n T_n(E)$

Chebyshev Polynomial Expansion of the density of states



box random potential
1000 sites, 1 sample
up to 1000th order

Thouless Formula

D.J. Thouless, J. Phys. C **5** (1972) 77

$$\kappa(E) = \int_{-1}^{1} \rho(E') \ln|E - E'| dE' - \ln|t_{\mathrm{h}}|$$

$$G_{0N}(E) = \frac{{t_{\mathrm{h}}}^N}{\det(EI - H)} = {t_{\mathrm{h}}}^N \prod_{\mu=1}^{N} \frac{1}{E - E_\mu}$$

$$\simeq e^{-\kappa(E)N}$$

Chebyshev Polynomial Expansion of the inverse localization length

N. Hatano & J. Feinberg, PRE 94, 063305 (2016)

$$\kappa(E) = \int_{-1}^{1} \rho(E') \ln|E - E'| dE' - \ln|t_{\mathrm{h}}|$$

$$\rho(E) = \frac{1}{\sqrt{1-E^2}} \sum_{n=0}^{\infty} c_n T_n(E)$$

$$\int_{-1}^{1} T_n(E') \ln|E - E'| \frac{dE'}{\sqrt{1-E'^2}} = -\frac{\pi}{n} T_n(E)$$

(n ≥ 1)

Chebyshev Polynomial Expansion of the inverse localization length

N. Hatano & J. Feinberg, PRE 94, 063305 (2016)

(i) $T_{n+1}(H) = 2HT_n(H) - T_{n-1}(H)$

(ii) $c_n = \dfrac{2}{N\pi} \operatorname{Tr} T_n(H)$

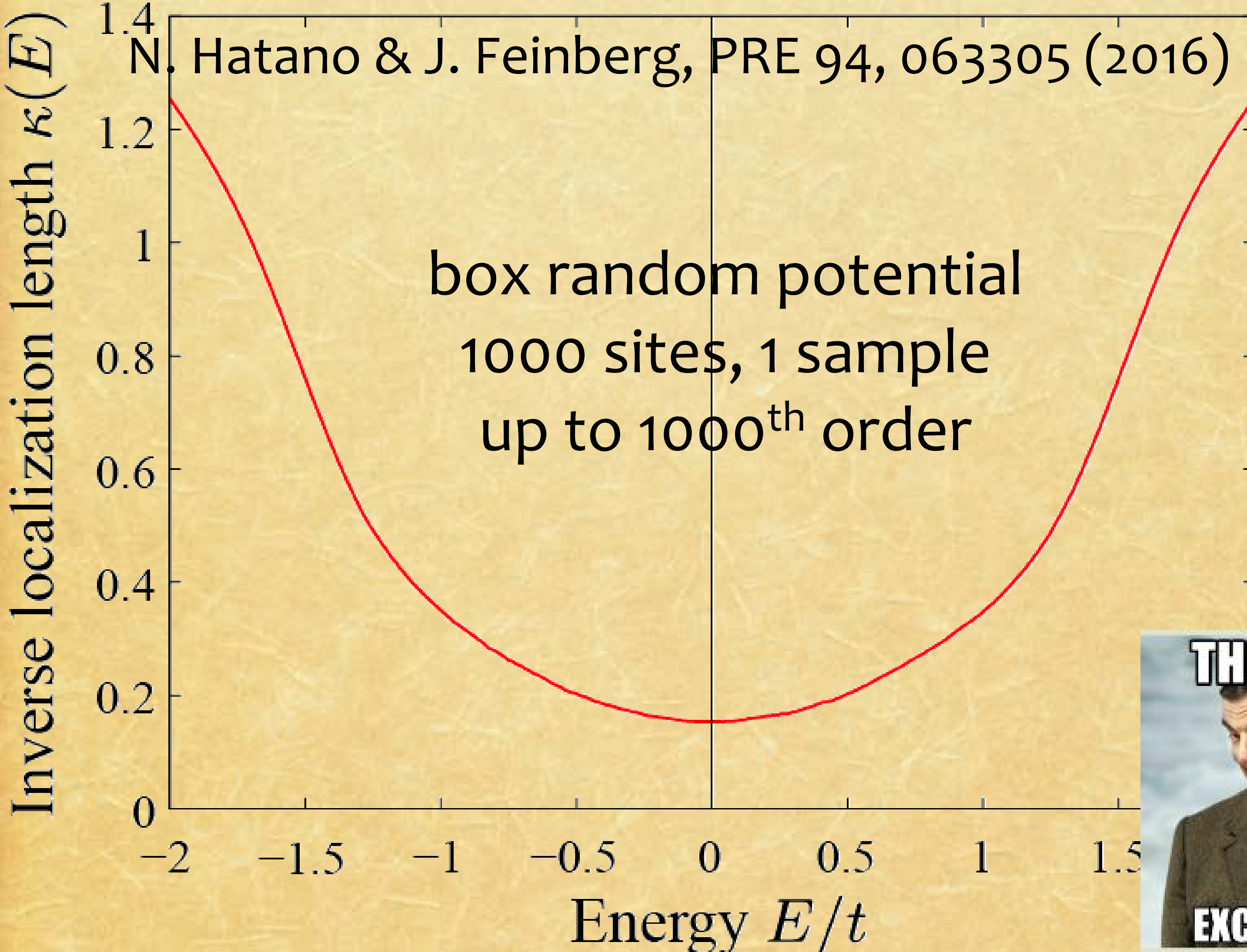
(iii) $\kappa(E) = -\pi \sum_{n=1}^{\infty} \dfrac{c_n}{n} T_n(E) - \ln 2|t_{\mathrm{h}}|$

cutoff (→ ∞)

better convergence (→ n)

Chebyshev Polynomial Expansion of the inverse localization length

N. Hatano & J. Feinberg, PRE 94, 063305 (2016)



box random potential
1000 sites, 1 sample
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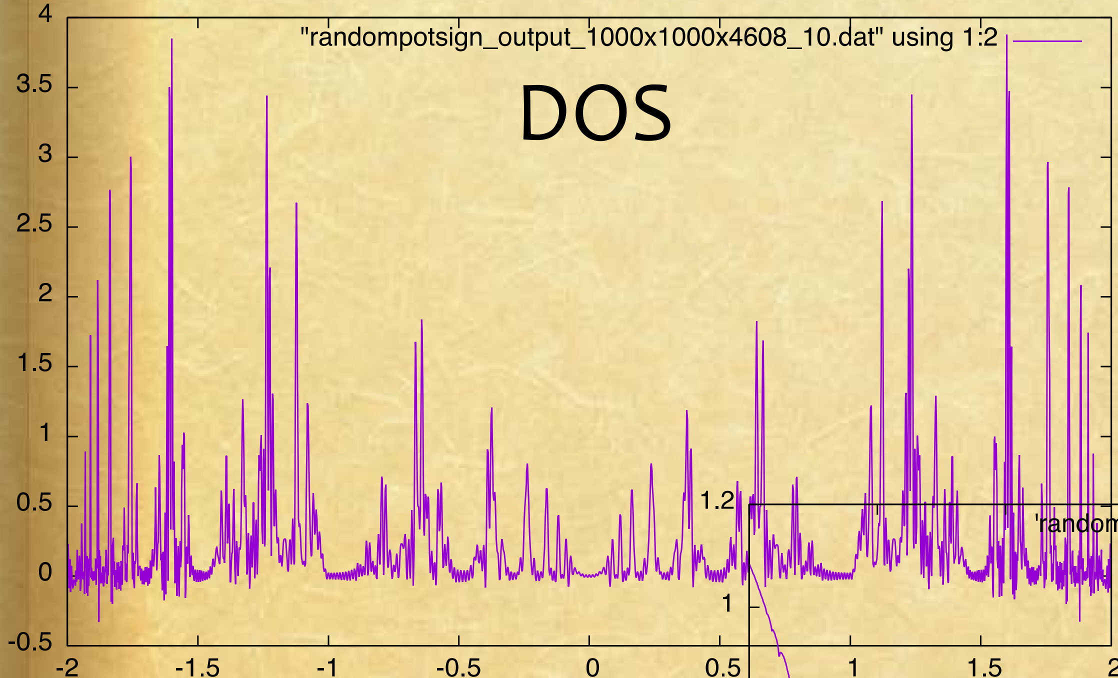
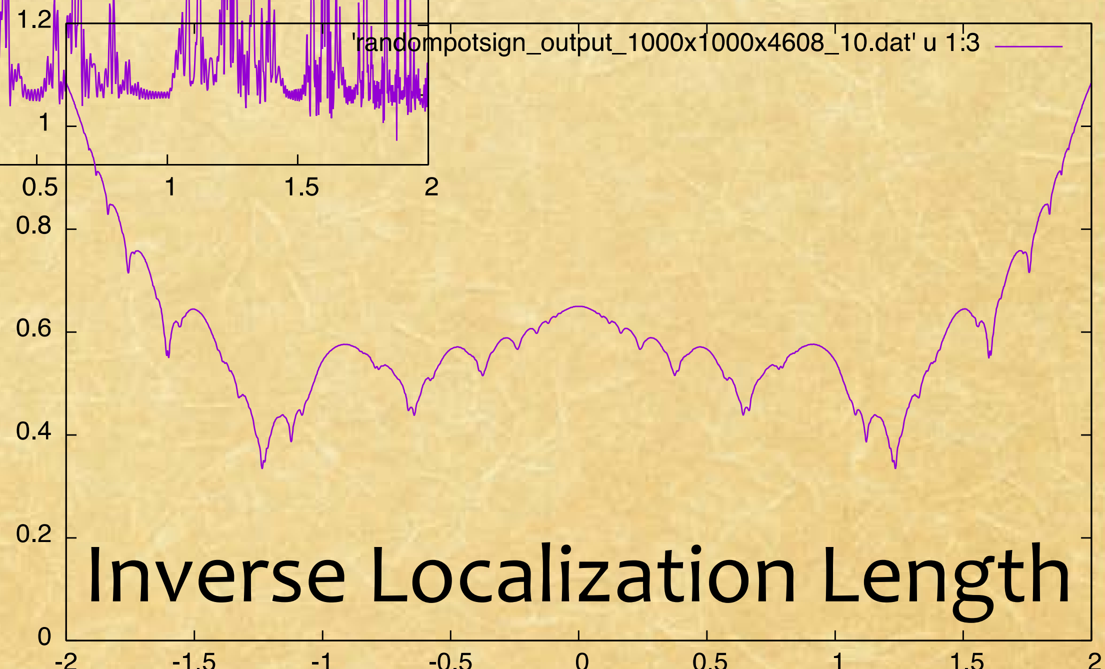
Random-Sign Potential





Summary

- Random-Sign Potential

Unconventional behavior?

- Chebyshev polynomial expansion

Inverse localization length