The Abdus Salam
International Centre
for Theoretical Physics

Random Matrices, Integrability and Complex Systems ,
Yad Hashmona, Israel
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NON-ERGODIC EXTENDED PHASE IN GENERALIZED ROSENZWEIG-PORTER RMT

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NJP 17, 12202, (2015)**,**
arXiv: 1805.06472,
arXiv: 1810.01492

Collaboration:

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Rosenzweig-Porter RMT

$$\langle H_{nm} \rangle = 0$$

$$\sigma = \frac{\lambda^2}{N^{\gamma}} << 1$$

Special diagonal:
Rosenzweig-Porter (1960) ensemble

NxN matrix, uncorrelated random entries

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

Simplest non-invariant RMT

Localization/delocalization sufficient conditions (for uncorrelated entries)

Convergence of Anderson's locator expansion

$$\frac{1}{N}\sum_{n,m=1}^{N}\langle |H_{nm}|\rangle < \infty, \quad \Rightarrow \text{ localized}$$

$$\langle |H_{nm}|\rangle \,\square\, \delta \sim N^{-1}$$

$$\frac{1}{N}\sum_{n,m=1}^{N}\left\langle |H_{nm}|^2\right\rangle = \infty \quad \Rightarrow \text{ semicircle}, \quad \Rightarrow \text{ ergodic extended}$$

Mott's criterion V>W

$$V = \sqrt{2S}$$

RP:

$$\left\langle (H_{nn})^2\right\rangle = W^2 = 1$$

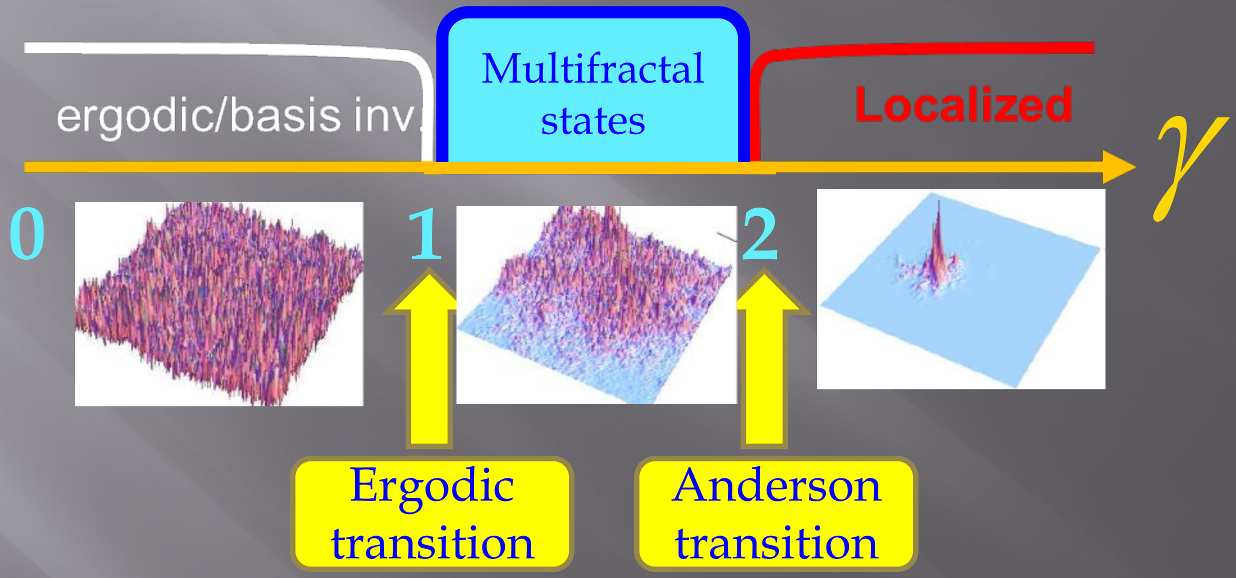
$$\left\langle |H_{n\neq m}|^2\right\rangle = \frac{\lambda^2}{N^{\gamma}}$$

$$\rho_0 = \frac{\sqrt{2S - E^2}}{\pi S}, \quad \gamma < 1$$

$$S = \frac{1}{N}\sum_{n,m=1}^{N}\left\langle |H_{nm}|^2\right\rangle = \lambda^2 N^{1-\gamma}$$

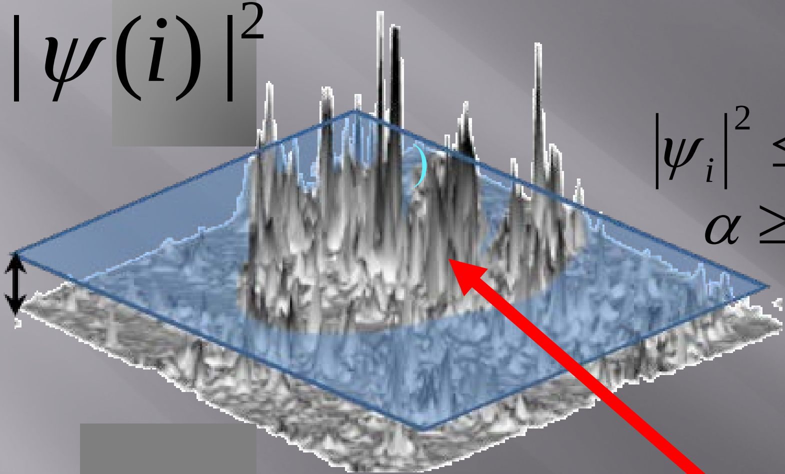
Ergodic transition

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini,
New J. Phys., v.17, 12202 (2015)



Crash course on multifractality

$|\psi(i)|^2$



$N^{-\alpha}$

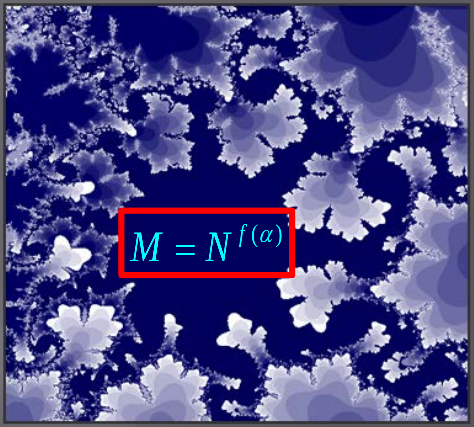
$|\psi_i|^2 \leq 1$

$\alpha \geq 0$

***N* sites {i} in the sample**

M sites in this region

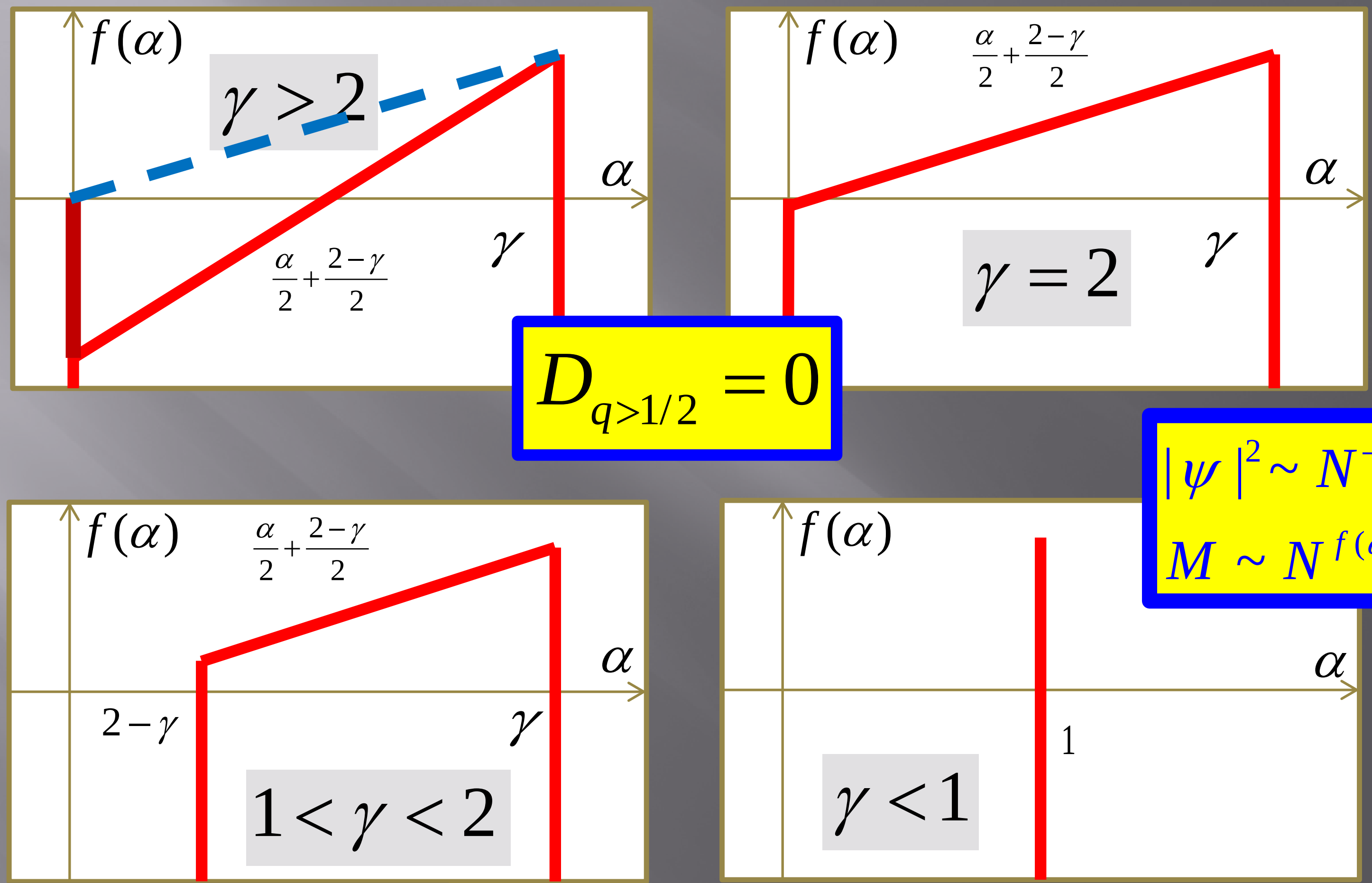
Map of the regions with amplitude larger than the chosen level



$M = N^{f(\alpha)}$

$$\sum_i |\psi(i)|^{2q} = \frac{c_q}{N^{D_q(q-1)}}$$

Multifractality spectrum f(α)



$$D_{q>1/2} = 0$$

$$|\psi|^2 \sim N^{-\alpha}$$
$$M \sim N^{f(\alpha)}$$

$$D_{q>1/2} = 2-\gamma$$

Fractal dimension of wavefunction support set

$$D_q = 1$$

Existence of multifractal phase and ergodic transition in RP RMT is suggested (with the physical standard of rigor) in:

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini, New J. Phys., v.17, 12202 (2015)

and rigorously proven (on the level of a math theorem):

Per von Soosten and S. Warzel, Electron J. Probab. 23, 1 (2018).

arXiv: 1709.10313 [math-ph)]

Ansatz for random wave functions of Rosenzweig-Porter RMT

$$|\psi_n(i)|^2 = \frac{|H_{ni}|^2}{(E_n - E_i)^2 + \Gamma(N)^2}$$

$$\delta(N) = (\rho_0 N)^{-1}$$

$$\Gamma(N) = \begin{cases} \delta(N) N^D & \text{extended states} \\ \sqrt{\langle |H_{n\neq m}|^2 \rangle}, & \text{localized states} \end{cases}$$

$$\rho_0 = p(E) \sim 1, \quad (\gamma > 1)$$

Semi-circle

$$\rho_0 = \frac{\sqrt{2S - E^2}}{\pi S}, \quad \gamma < 1$$

$$S = \sum_{n,m=1}^{N} \langle |H_{nm}|^2 \rangle = \lambda^2 N^{1-\gamma}$$

$$\Gamma(N) = \begin{cases} N^{\frac{1-\gamma}{2}}, & (\gamma < 1, \text{ EE}) \\ N^{-(\gamma-1)}, & (1 < \gamma < 2, \text{ NEE}) \\ N^{-\gamma/2}, & (\gamma > 2, \text{ L}) \end{cases}$$

$\Gamma \to \infty$

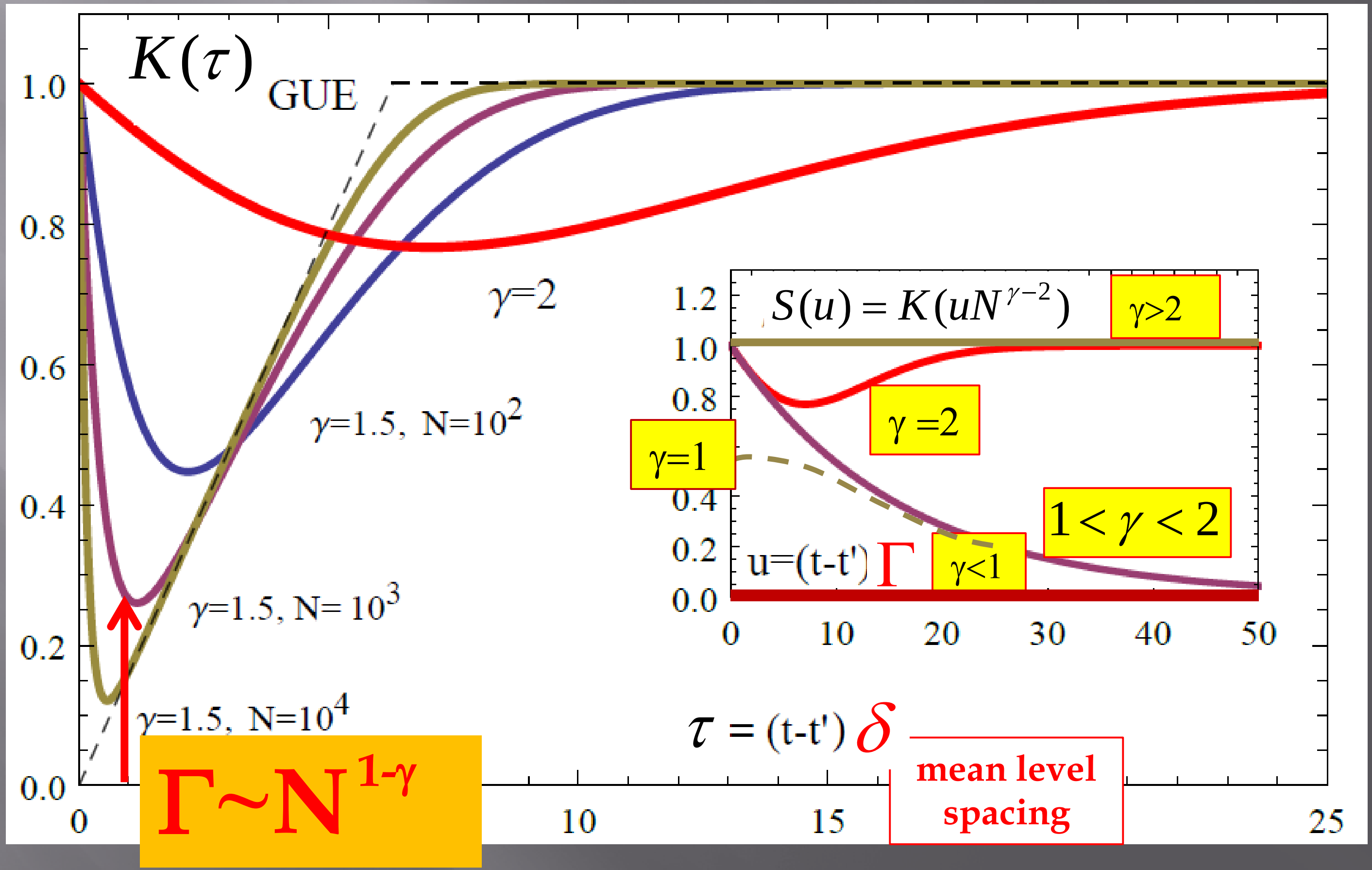
$\Gamma \to 0, \ \Gamma/\delta \to \infty$

$\Gamma \to 0, \ \Gamma/\delta \to 0$

Spectral form-factor and the ‘hybrid’ level statistics

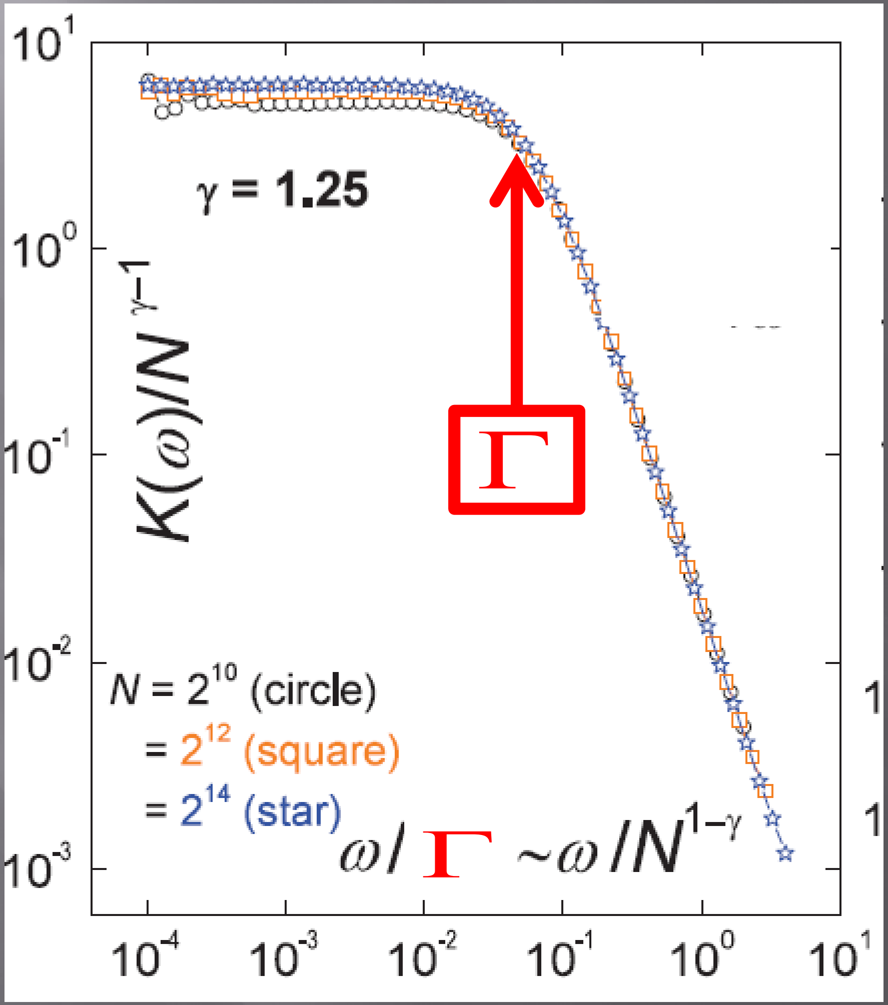
$$S(u) = 1 + e^{-2\pi\Lambda^2 u} e^{-\Lambda^2 u^2 N^{\gamma-2}} \left[\frac{2I_1(\kappa u^{3/2})}{\kappa u^{3/2}} - \frac{1}{4\pi}\kappa u^{5/2} N^{\gamma-2} \int_0^\infty \frac{x\,dx}{\sqrt{x+1}} I_1(\kappa u^{3/2}\sqrt{x+1})\, e^{-x\,u^2\Lambda^2 N^{\gamma-2}} \right]$$

$$\kappa = \sqrt{8\pi N^{\gamma-2}}\Lambda^2 \text{ and } \Lambda = \lambda p(0)$$



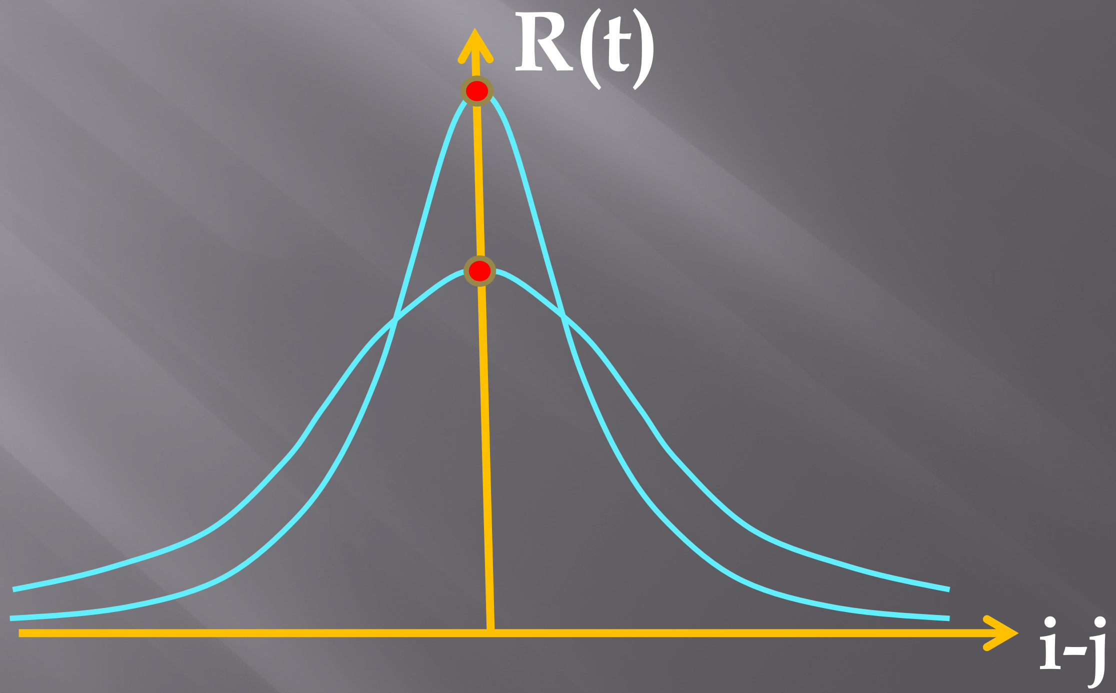
How to detect the new scale Γ?

$$K(\omega)=\sum_{\alpha,\beta}|\psi_\alpha(i)|^2|\psi_\beta(i)|^2\delta(\omega-E_\alpha+E_\beta)$$

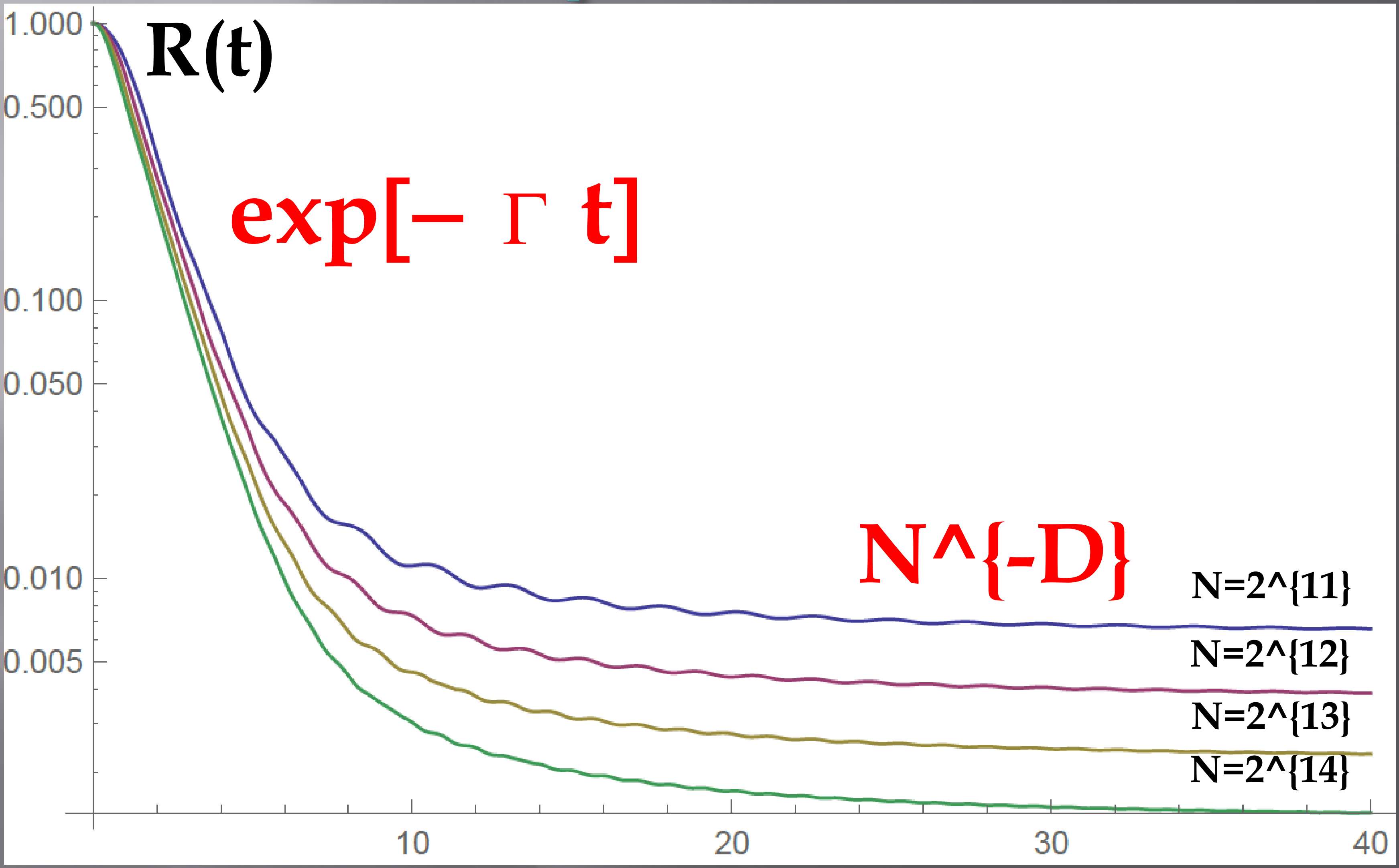


More informative Fourier transform: Survival Probability

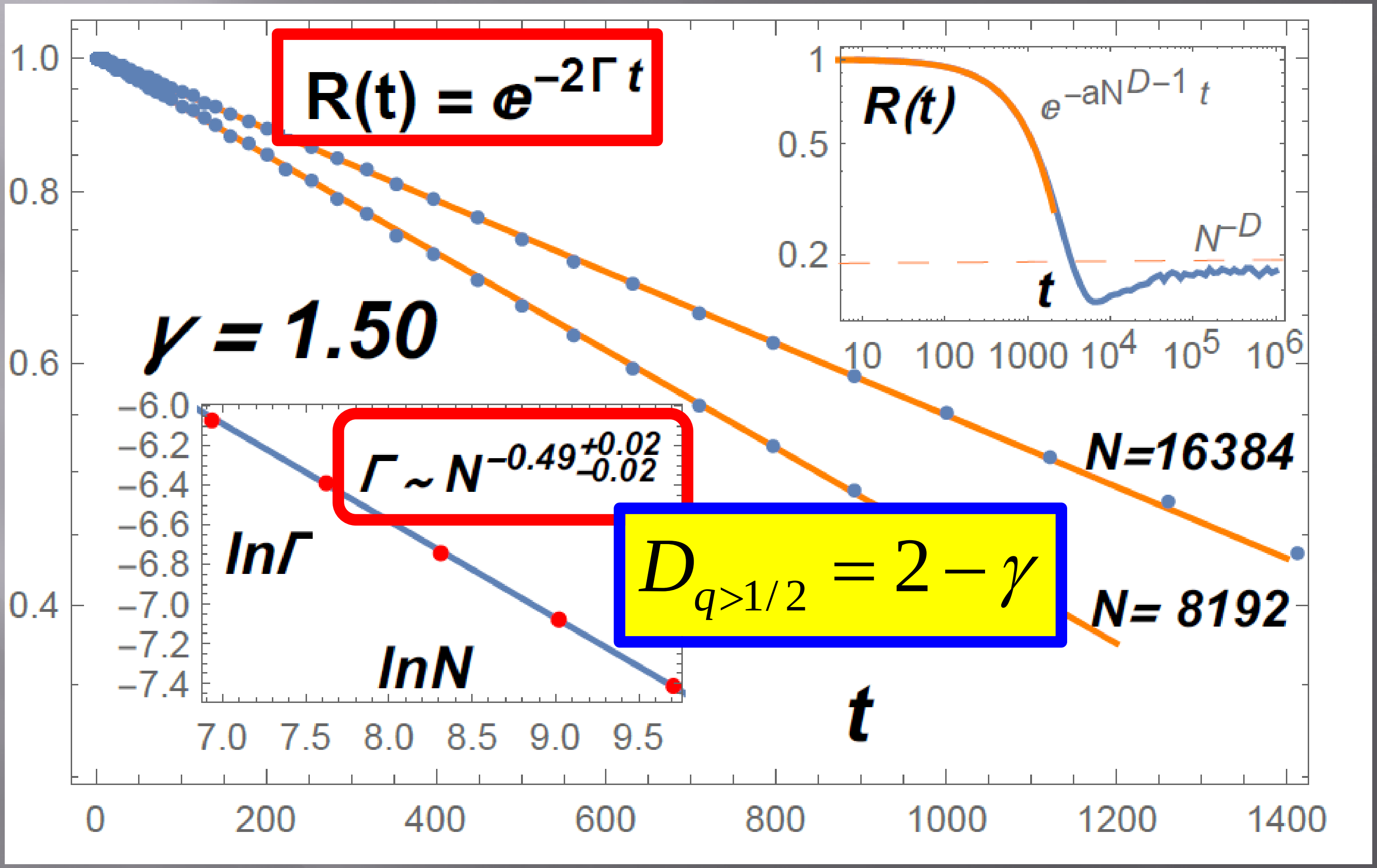
$$R(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2$$
$$= \sum_{\alpha,\beta} |\psi_\alpha(i)|^2 |\psi_\beta(i)|^2 \exp\left[i(E_\alpha - E_\beta)t \right]$$



Survival Probability in MF phase



Survival probability in NEE phase



What about correlated hopping terms?

Exactly soluble: all states are localized (γ>2) or critically localized (γ<=2)

Yuzbashyan-Shastry (YS) model

$$H_{n\neq m} = g_n g_m^*$$

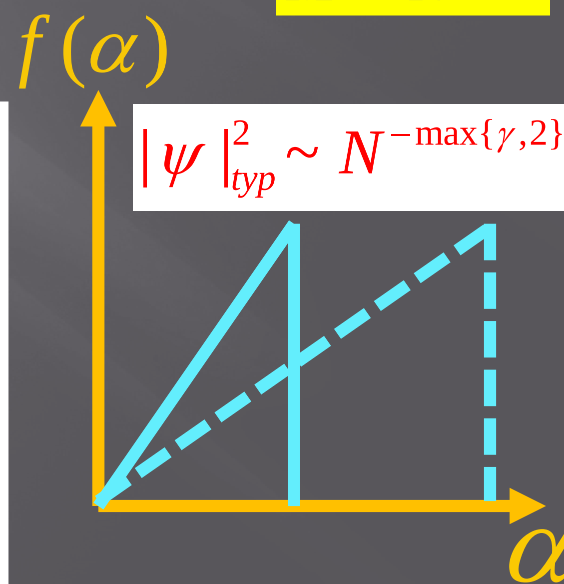
$$g_n = \frac{1}{N^{\gamma/2}}$$

$$\Psi_E(i) = c\,\frac{g_i}{E-\varepsilon_i}$$

$$\sum_{i=0}^{N-1} \frac{g_i^2}{E-\varepsilon_i} = -1$$

$$|\psi|^2 \sim N^{-\alpha}$$

$$M \sim N^{f(\alpha)}$$



R. Modak, S. Mukerjee, E. A. Yuzbashyan, and B. S. Shastry, New J. Phys. **18**, 033010 (2016).
H. K. Owusu, K. Wagh, and E. A. Yuzbashyan, J. Phys. A: Math. Theor. **42**, 035206 (2009).
A. Ossipov, J. Phys. A **46**, 105001 (2013).
G. L. Celardo, R. Kaiser, and F. Borgonovi, Phys. Rev. B **94**, 144206 (2016).
X. Deng, V. E. Kravtsov, G. V. Shlyapnikov, and L. Santos, Phys. Rev. Lett. **120**, 110602 (2018).

Why no delocalized states?

Convergence of Anderson's locator expansion

$$\frac{1}{N}\sum_{n,m=1}^{N}\langle |H_{nm}|\rangle < \infty, \quad \Rightarrow \text{ localized}$$

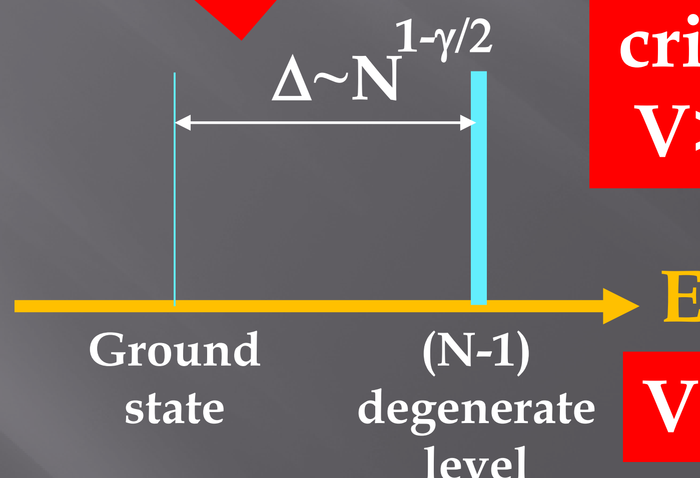
$$\langle |H_{nm}|\rangle \sim \delta \sim N^{-1}$$

$$\frac{1}{N}\sum_{n,m=1}^{N}\langle |H_{nm}|^2\rangle = \infty \quad \Rightarrow \text{ ...rgodic extended}$$

Not valid for correlated entries

Mott's criterion V>W=1

However, Mott's criterion is valid!



V=0

Not only YS!

No delocalized states also for deterministic power-law hopping!

$$H_{nm} = J\,|n-m|^{-a}$$

Even for a < 1

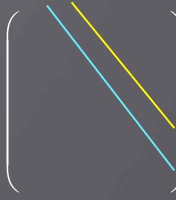
P. Nosov, I.M.Khaymovich and V.E.K. arXiv: 1810.01492

X. Deng, V.E.K. G. Shlyapnikov, L. Santos,
Phy. Rev. Lett. **120**, 110602 (2018)

Translation-invariant RP model: correlation along the (non-principle)diagonal

$$H_{n\neq m} = H_{n-m}$$

$$\langle H_{n-m}\rangle = 0,$$
$$\langle |H_{n-m}|^2\rangle = N^{-\gamma}$$



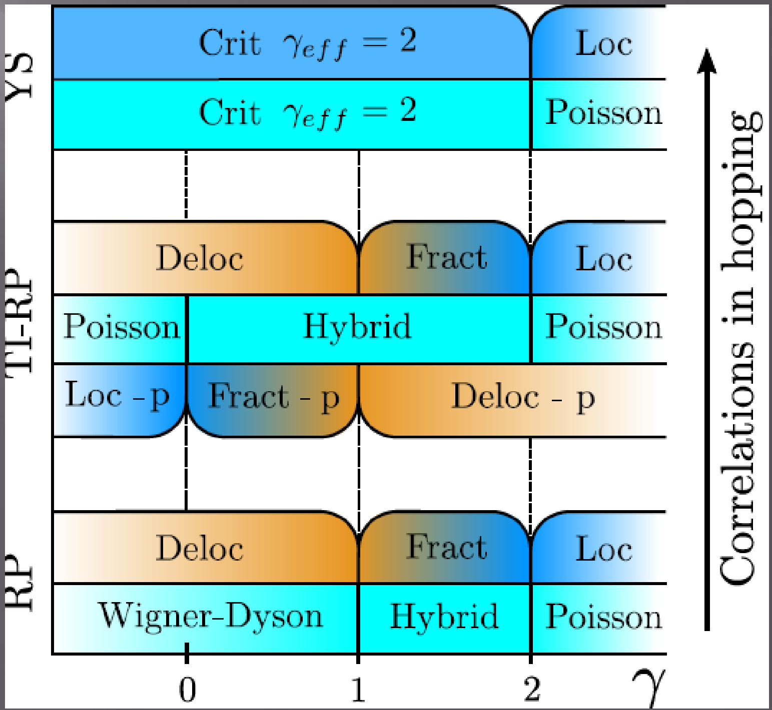
Correlated along the diagonal, Uncorrelated with zero mean between the diagonals

P. Nosov, I.M.Khaymovich and V.E.K. arXiv: 1810.01492

- ▷ Lack of correlations between diagonals destroy localization in the coordinate space for $\gamma<2$.
- ▷ Localization and multifractality in the momentum space

$$\gamma_p = 2-\gamma$$

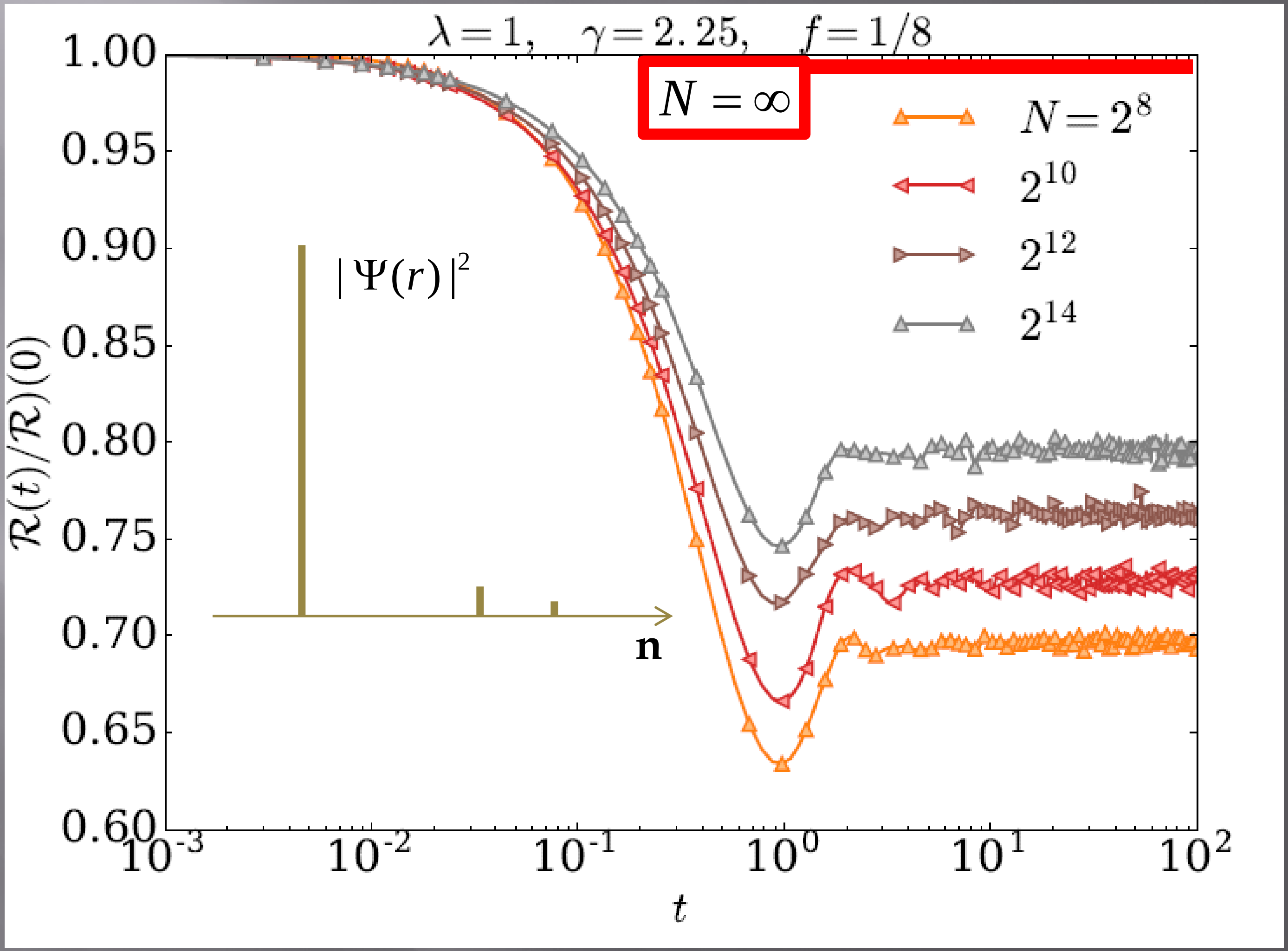
- ▷ Poisson and 'hybrid' level statistics in the delocalized phase



Conclusion

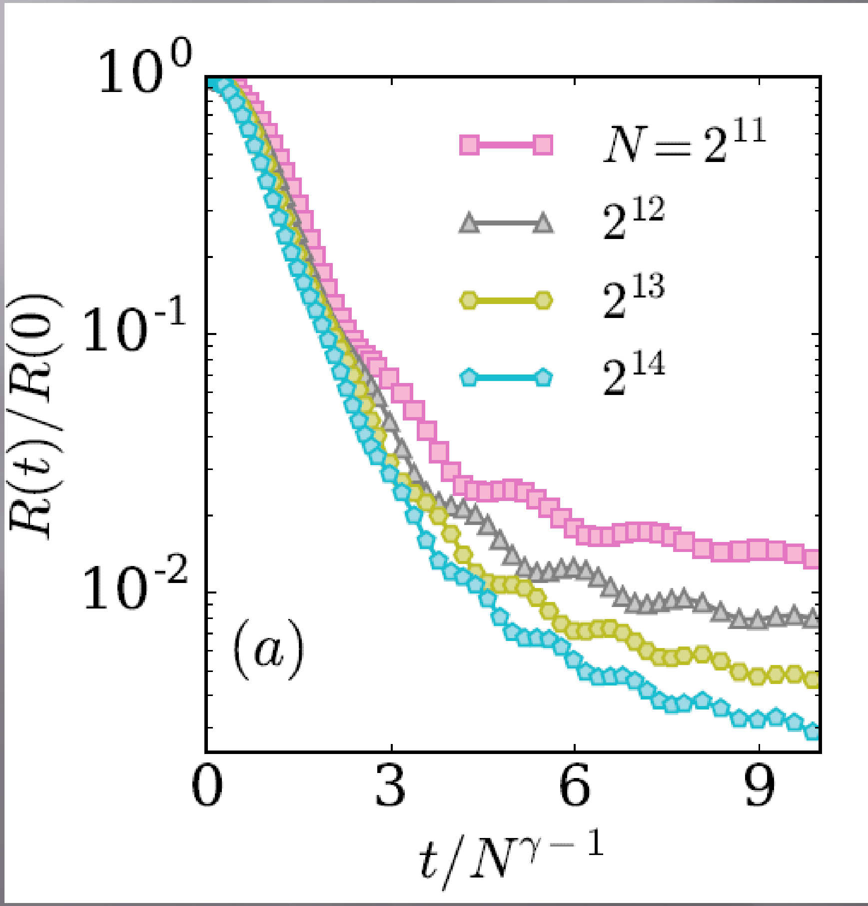
- Two extended phases and ergodic transition in RP RMT
- Ansatz for random wave functions of RP RMT and survival probability
- ‘Hybrid’ level statistics
- Localization in YS exactly solvable model: RP with fully correlated hopping
- Translation-invariant TI-RP: localization and multifractality in the momentum space; Poisson and ‘hybrid’ level statistics in delocalized phase

Survival probability in the localized phase of RP



Oscillations in NEE phase

$$P(t)-P(\infty)\propto e^{-2\pi|k|}+\frac{1}{2c\,(\pi k)^2}-\frac{\cos(2\pi kc)}{2(\pi k)^2\,c\,(c^2+1)}$$

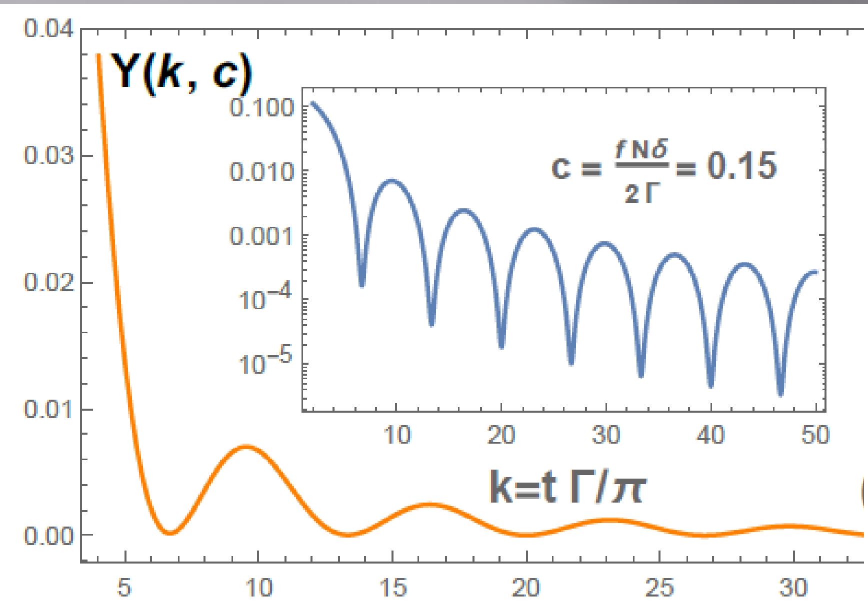


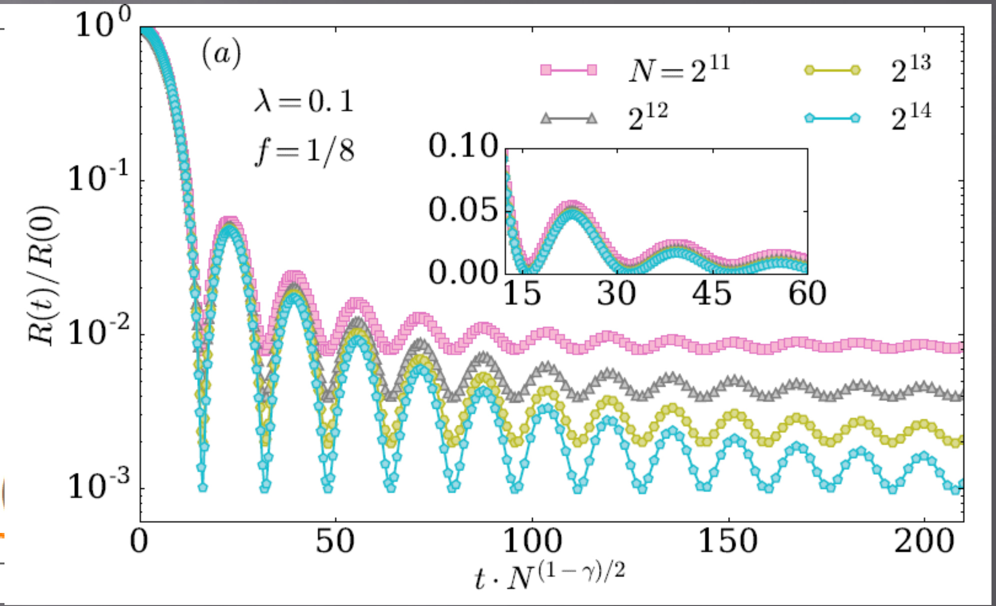
$$k=\frac{t\Gamma}{\pi}$$

$$c\propto\begin{cases}1, & \text{ergodic}\\ N^{\gamma-1}, & \text{NEE}(1<\gamma<2)\end{cases}$$

In NEE phase oscillations are a finite-size effect

Survival probability in EE phase





Oscillations in ergodic extended phase due to edge of DoS (semi-circle or finite fraction of states f)