

Stability and prethermalization in chains of classical kicked rotors

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Abstract

Periodic drives are a common tool to control physical systems, but have a limited applicability because time-dependent drives generically lead to heating. How to prevent the heating is a fundamental question with important practical implications. We address this question by analyzing a chain of coupled kicked rotors, and find two situations in which the heating rate can be arbitrarily small: (i) linear stability, for initial conditions leading to an effective integrability, and (ii) marginal localization, for drives with large frequencies and small amplitudes. In both cases, we find that the dynamics shows universal scaling laws that allow us to distinguish localized, diffusive, and sub-diffusive regimes. The marginally localized phase has common traits with recently discovered pre-thermalized phases of many-body quantum-Hamiltonian systems, but does not require quantum coherence.

Coupled kicked rotors

$$H = \sum_{j=1}^N \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

Classical dynamics

$$p_j(n+1) = p_j(n) - \kappa [\sin(\phi_j(n) - \phi_{j+1}(n+1)) - \sin(\phi_{j-1}(n) - \phi_j(n+1))]$$

$$\phi_j(n+1) = \phi_j(n) + p_j(n+1)\tau$$

Average of square of momentum

$$\langle p^2 \rangle = \frac{1}{N} \langle \sum_{j=1}^N p_j^2 \rangle$$

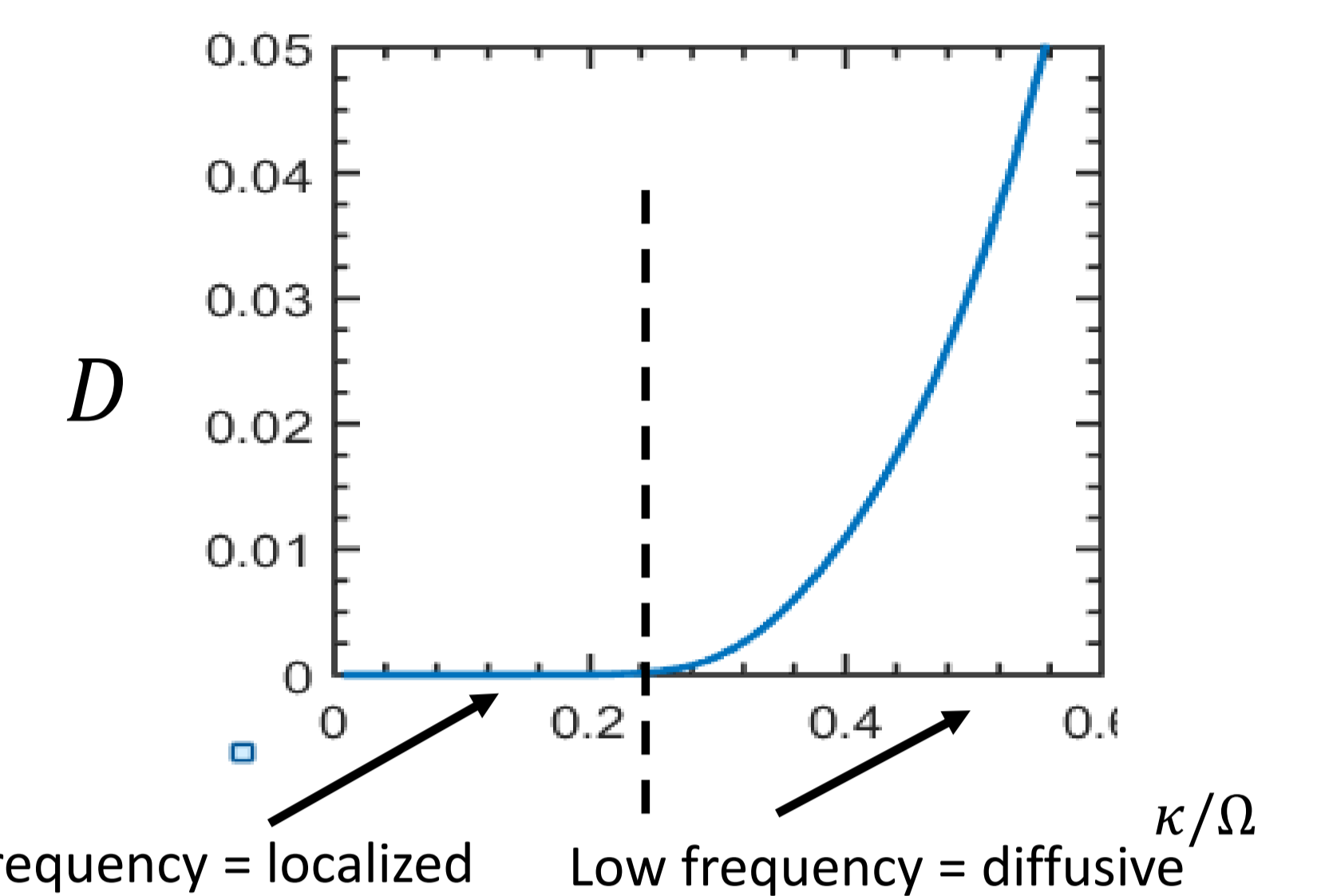
Always diffusive
Kaneko & Konishi (1989)
Chirikov & Vecheslavov (1997)

Diffusion coefficient

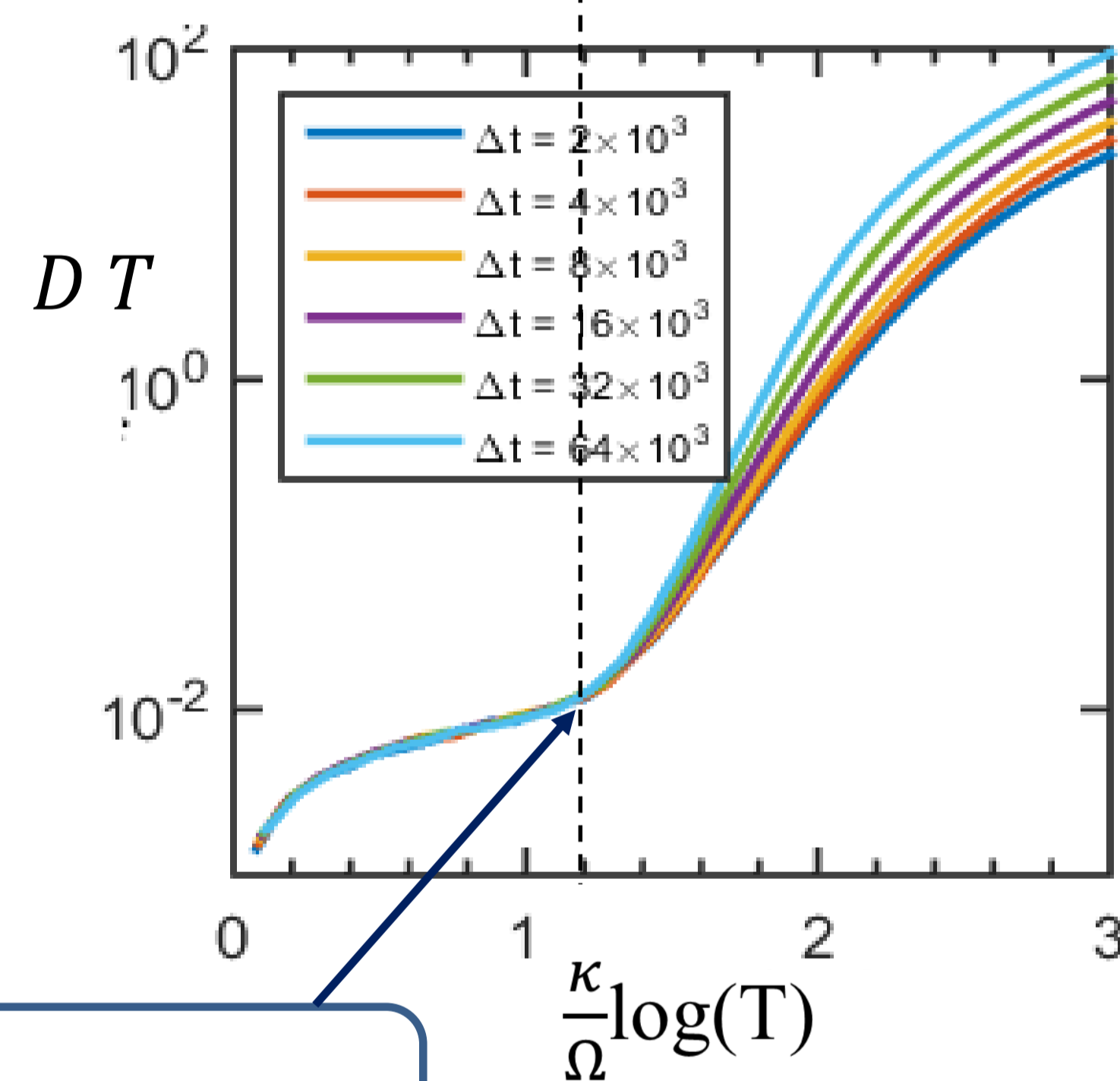
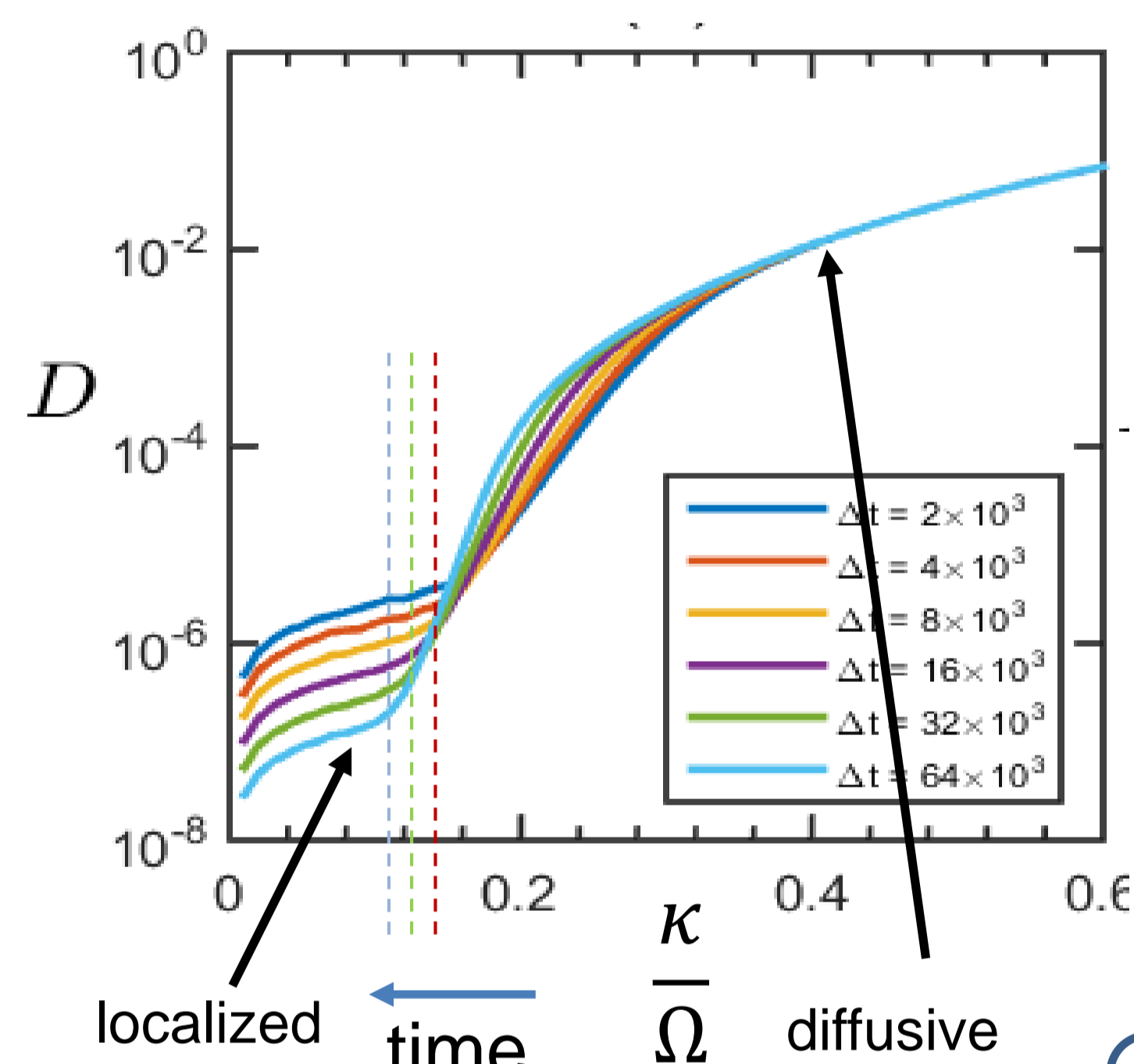
$$D = \frac{\langle p(T)^2 \rangle - \langle p(0)^2 \rangle}{T}$$

Diffusion exponent (α)

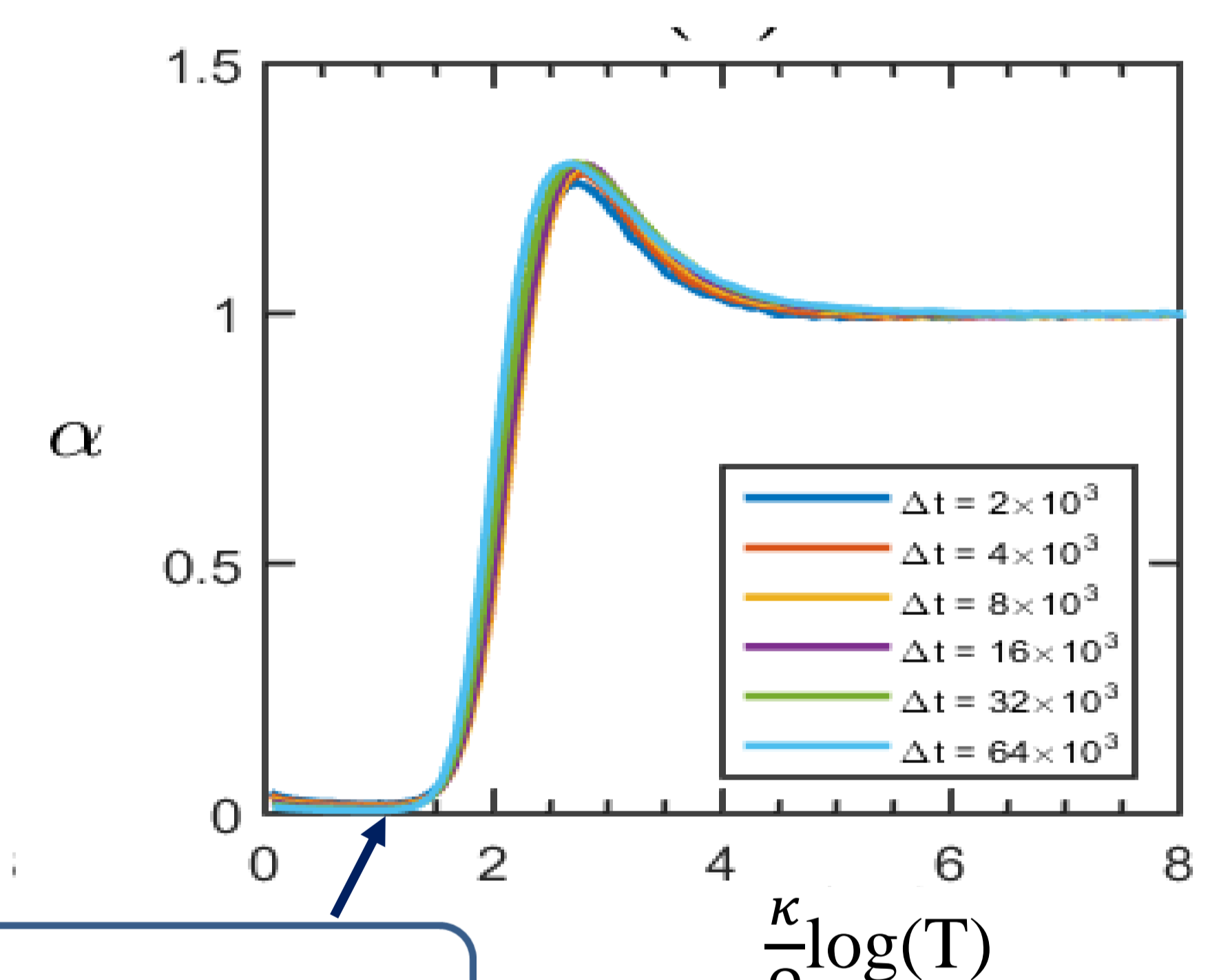
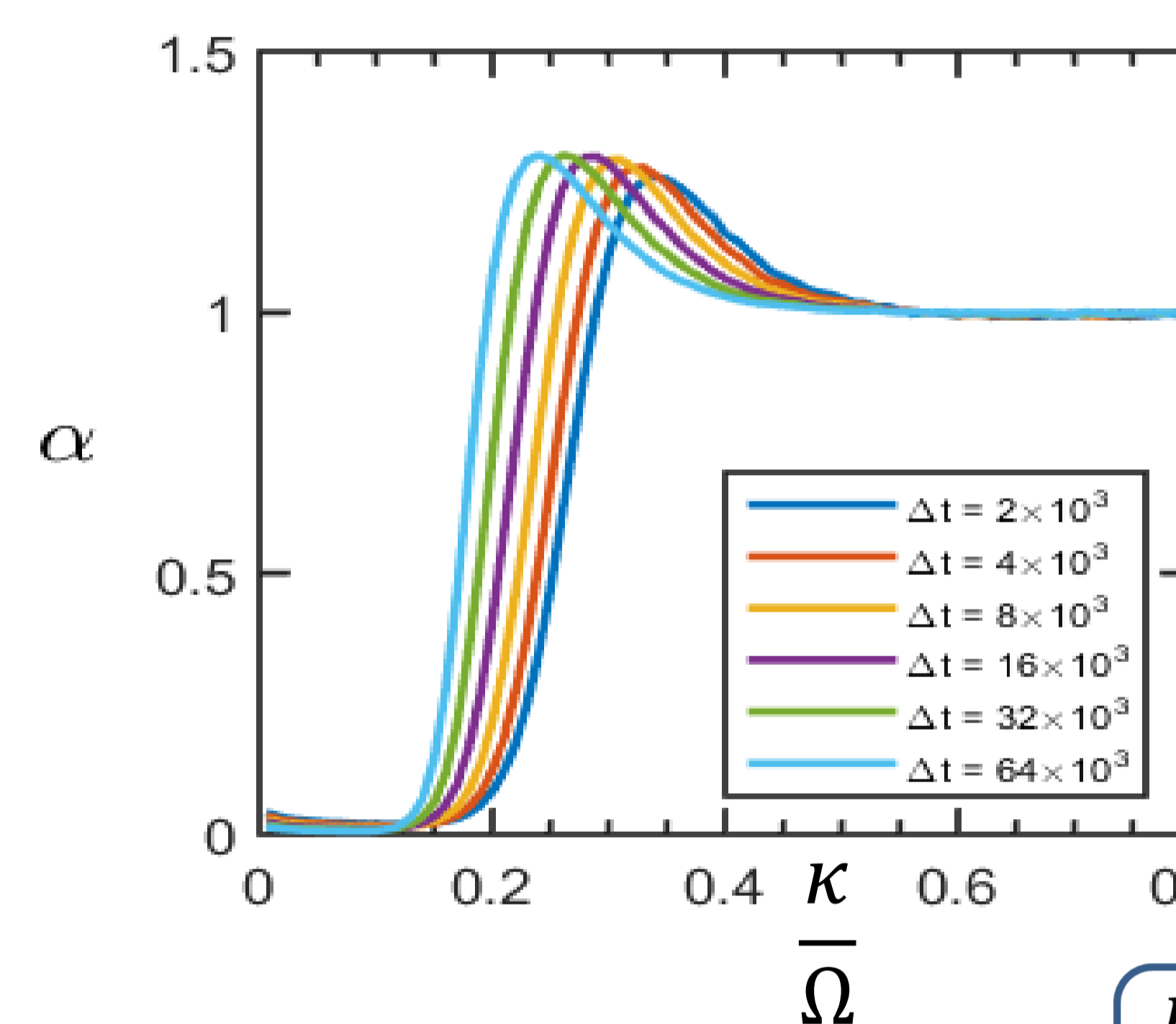
$$\langle p(t)^2 \rangle = A t^\alpha$$



Scaling analysis



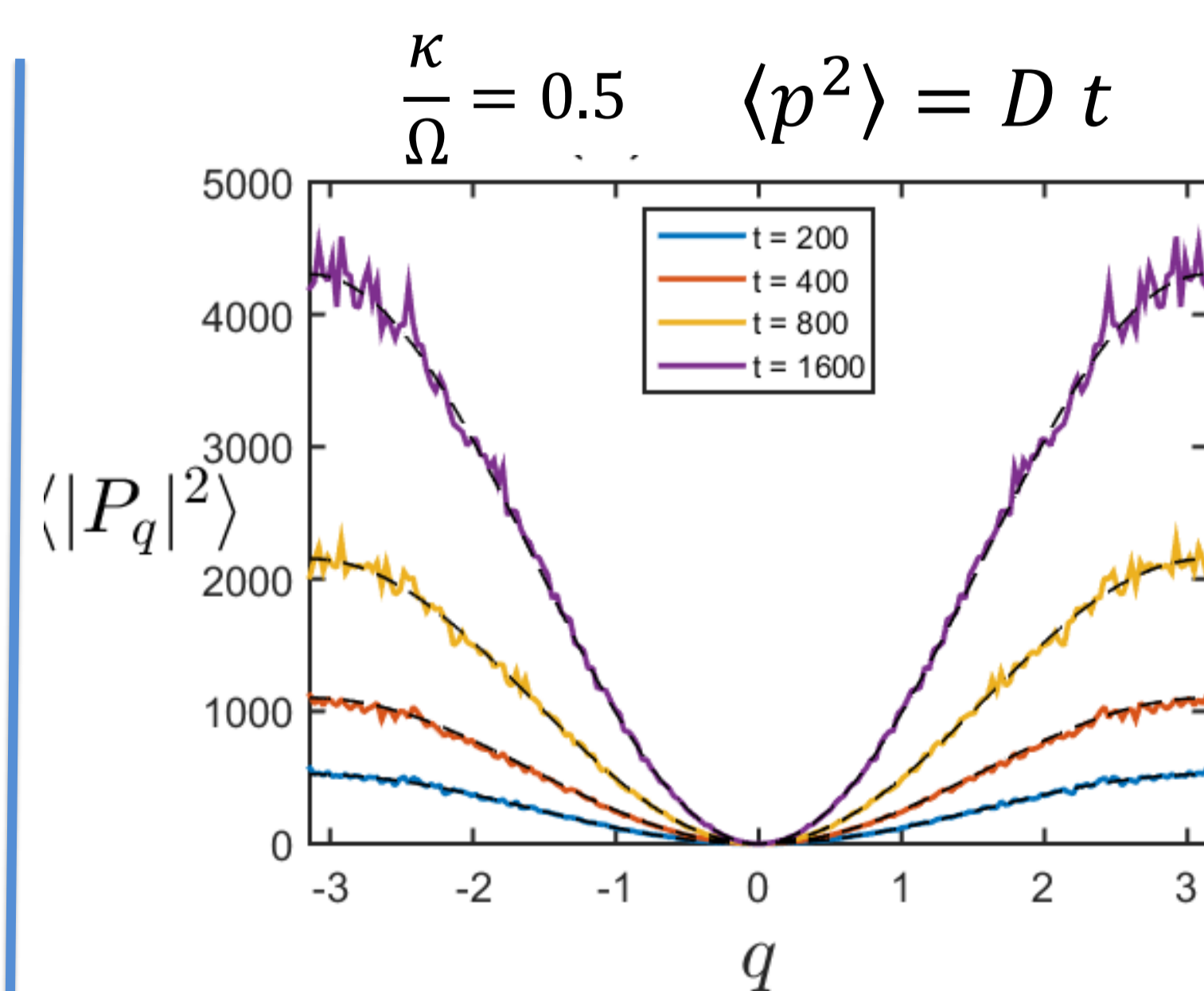
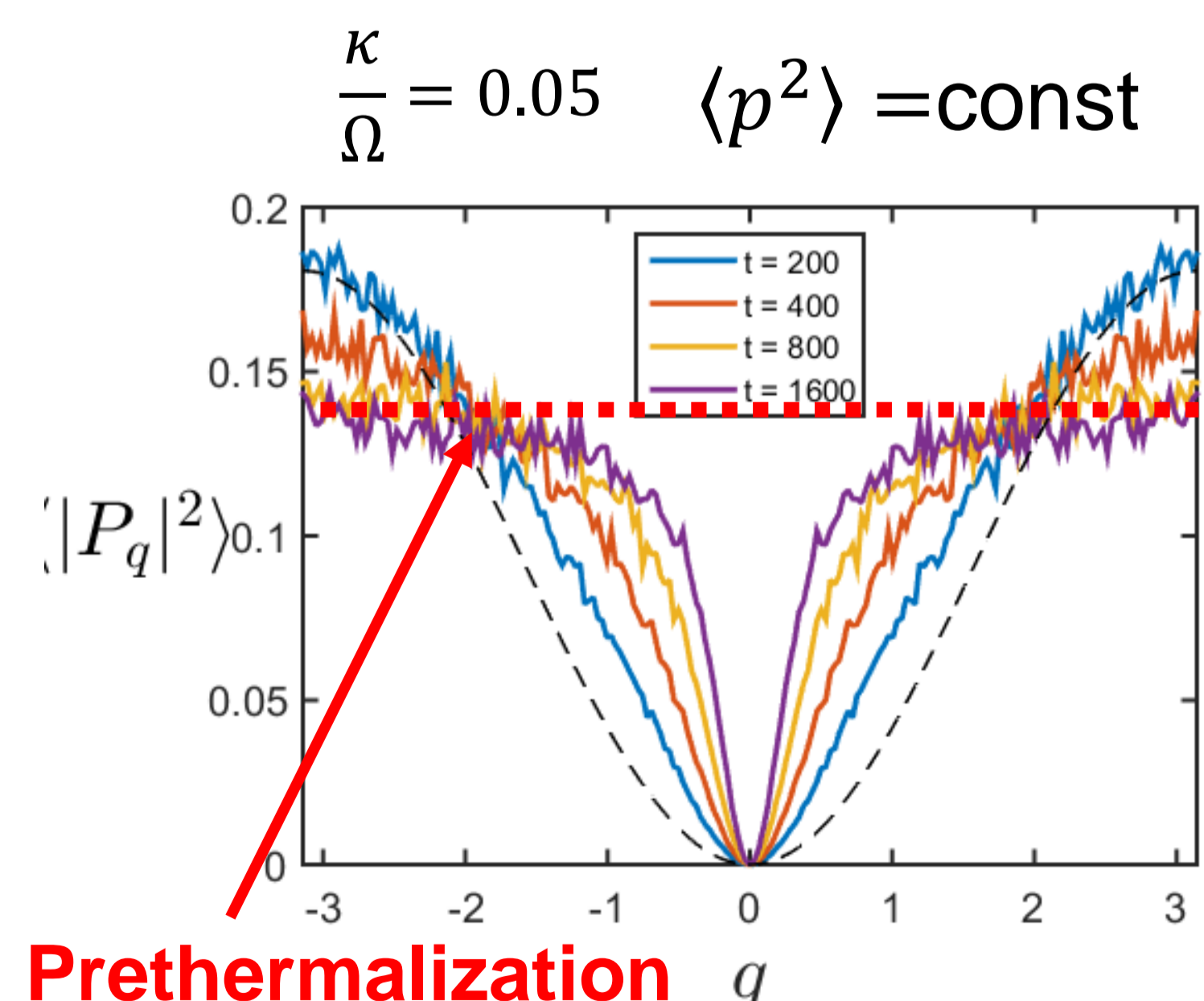
$$\frac{\kappa}{\Omega} \log(T) = 1.2$$



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Marginal localization

$$\frac{\kappa}{\Omega} \log(T^*) = 1.2 \Rightarrow T^* = \exp\left(\frac{1.2\Omega}{\kappa}\right)$$



Quadratic expansion

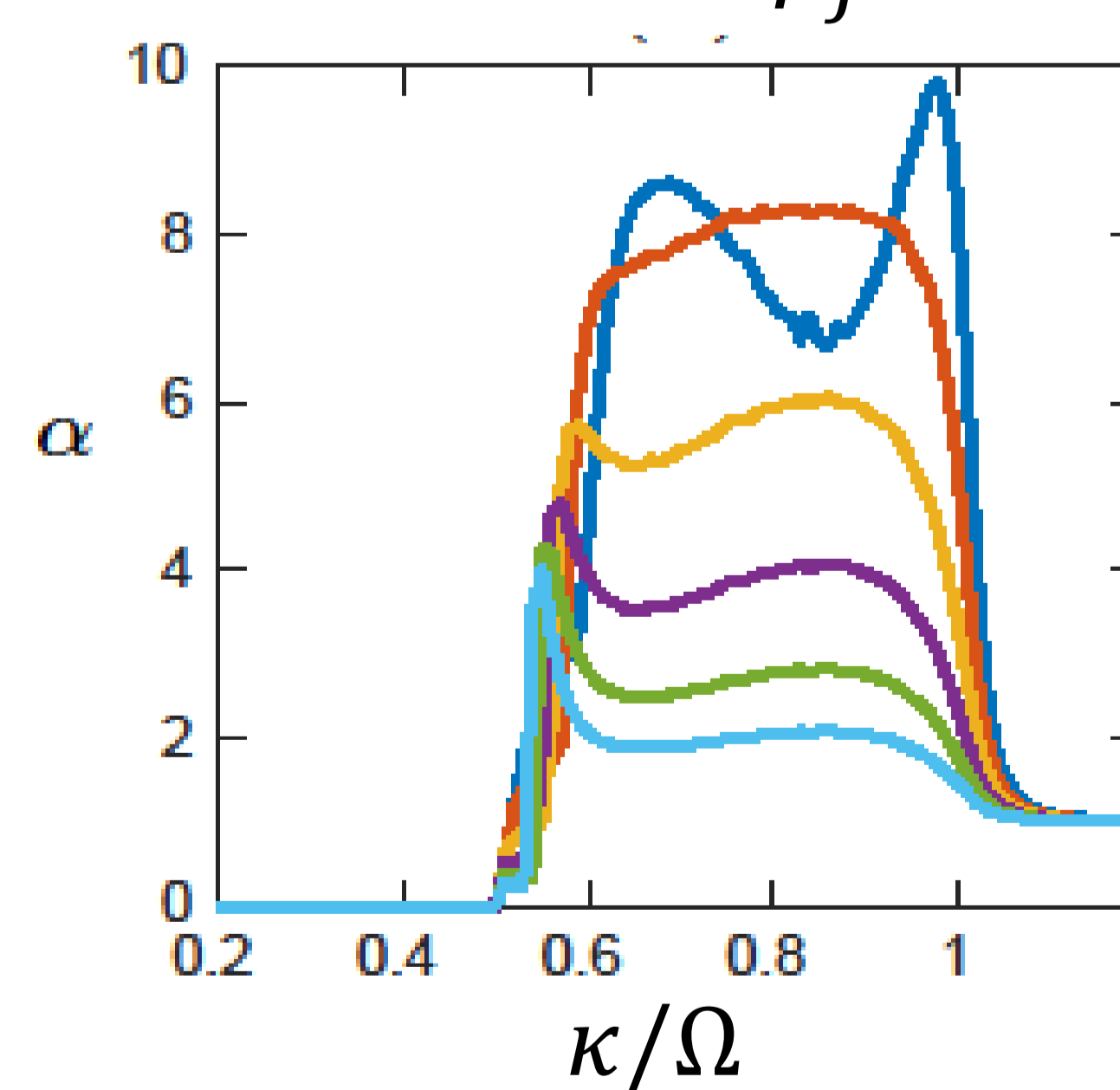
$$H = \frac{1}{2} \sum_q [|P_q|^2 + K(q) |\phi_q|^2 \sum_{n=-\infty}^{\infty} \delta(t - n\tau)]$$

$$K(q) = \frac{4\kappa}{\Omega} \sin^2\left(\frac{q}{2}\right)$$

→ Transition at $\kappa/\Omega = 1$

Quadratic instability

Initial state: $\phi_j \approx 0$

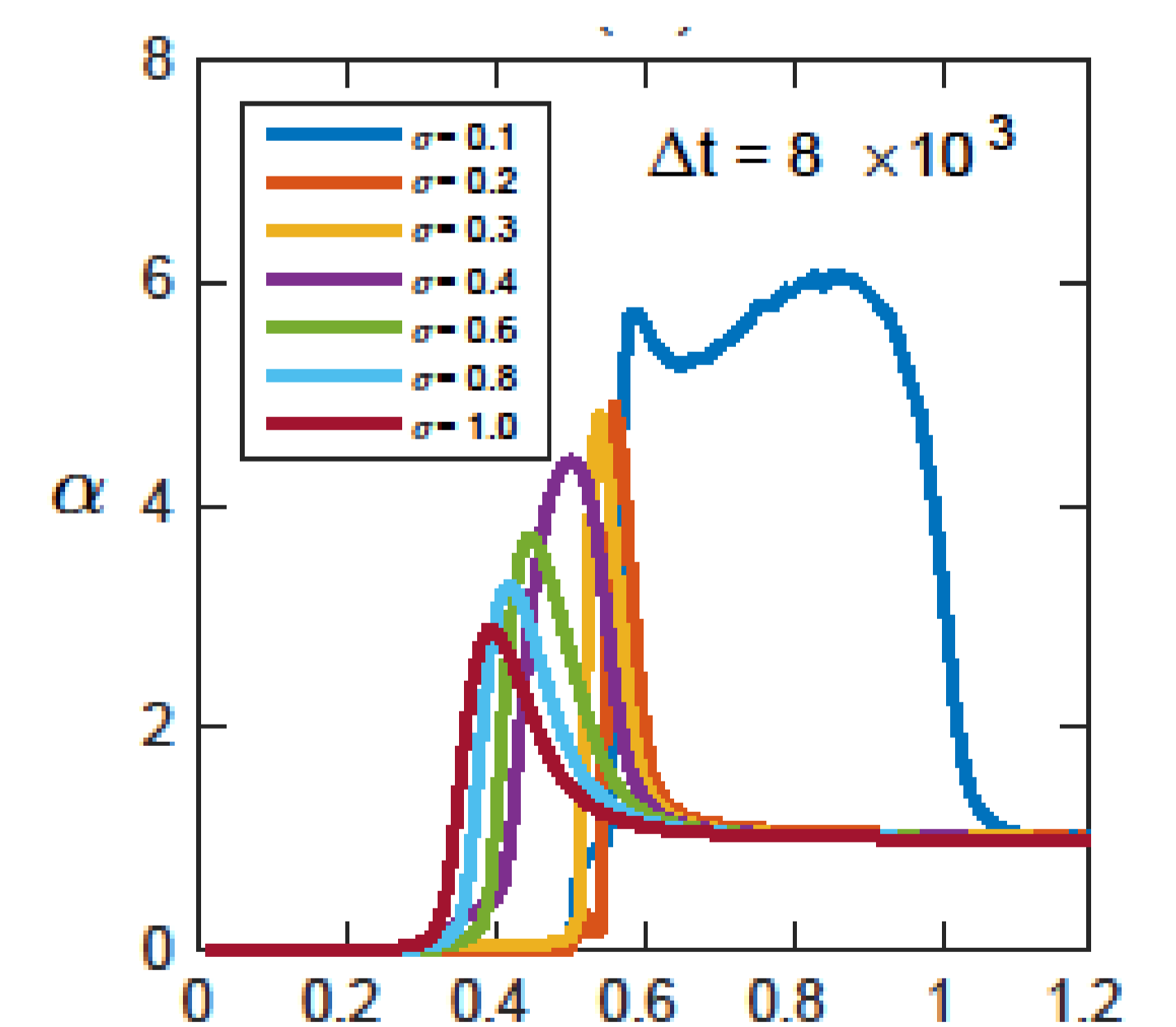


$$H^4 = -\frac{\kappa}{24} \sum_{j=1}^N (\phi_j - \phi_{j+1})^4 \sum_{m=1}^{\infty} \cos\left(\frac{2\pi m}{\tau} t\right),$$

$$2\pi m \leq 4 \cos^{-1}(1 - 2\kappa/\Omega)$$

$$m = 1 \Rightarrow \frac{\kappa}{\Omega} > 0.5$$

Quadratic to marginal



Effect of higher order resonances

Ref:

A. Rajak, R. Citro, and E. G. Dalla Torre, arXiv: 1801.01142 (2018).