SYK model with quadratic perturbations: the route to a non-Fermi-liquid

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Outline

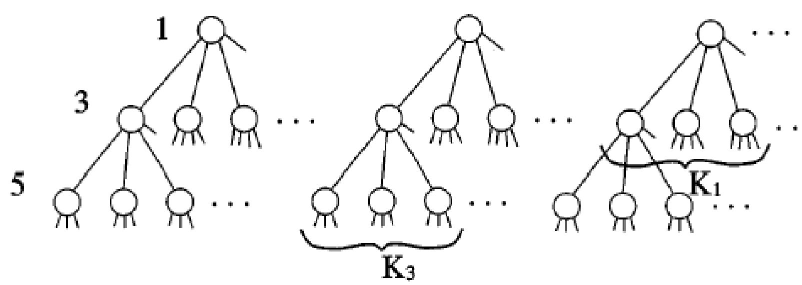
- Motivation
- SYK_4 model: low-energy effective theory
- $\mathrm{SYK}_4 + \mathrm{SYK}_2$ model: previous results
- Green function in $\mathrm{SYK}_4{+}\mathrm{SYK}_2$: perturbative calculation and numerics
- Conclusions

MBL in a quantum dot

$$H = \sum_\alpha \epsilon_\alpha c^+_\alpha c_\alpha + \sum_{\alpha\beta\gamma\delta} V^{\alpha\beta}_{\gamma\delta} c^+_\gamma c^+_\delta c_\beta c_\alpha$$

$V \sim \frac{\Delta}{g}$, $g \gg 1$ is conductance of the dot (internally screened Coloumb)

B. Altshuler, Y. Gefen, A. Kamenev, and L. Levitov (1997)



Typical many-body state of energy E has $N \sim \sqrt{E/\Delta}$ quasiparticles, with characteristic energy of each particle $T = \sqrt{E\Delta}$

Delocalization in the Fock space with $E \uparrow$

$$E_c \sim g^{2/3}\Delta$$

Gornyi, Mirlin, Polyakov, Burin (2017)

Numerics

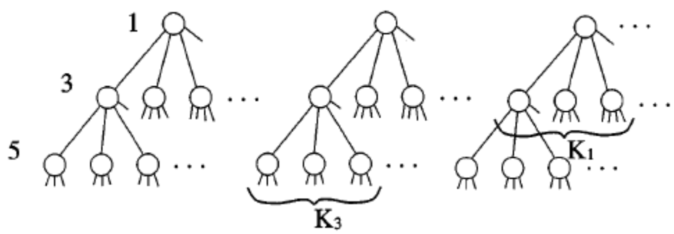
Jacquod, Shepelyansky (1997)
Georgeot, Shepelyansky (1997)
Leyronas, Silvestrov, Beenakker (2000)
Jacquod, Varga (2002)
Rivas, Mucciolo, Kamenev (2002)

Modelling Fock space

GOE can meaningfully be used in predicting spectral fluctuation properties of nuclei and other systems governed by two–body interactions (atoms and molecules). Nonetheless, embedded ensembles rather than GRTM would offer the proper way of formulating statistical nuclear spectroscopy. Unfortunately, an analytical treatment of the embedded ensembles is still missing.

Guhr, Müller-Groeling, Weidenmüller, 1997

Interacting problem: hopping over certain hierarchical sparse lattice



Cayley tree? NO: boundary is absent in the FS

Random Regular Graph (*aka* SRM)? BETTER, but number of fluctuators is very different: $\ln \mathcal{D}$ vs $\mathcal{D}$

Can SYK be a good starting point?

Sachdev-Ye-Kitaev model

Majorana Fermions χ_i satisfy $\{\chi_i, \chi_j\} = \delta_{ij}, \quad i, j = 1, ..., N$

SYK Hamiltonian: $H = \frac{1}{4!} \sum_{i,j,k,l} J_{i,j,k,l} \chi_i \chi_j \chi_k \chi_l$ with $\langle J_{ijkl}^2 \rangle = \frac{3! J^2}{N^3}$

This talk:

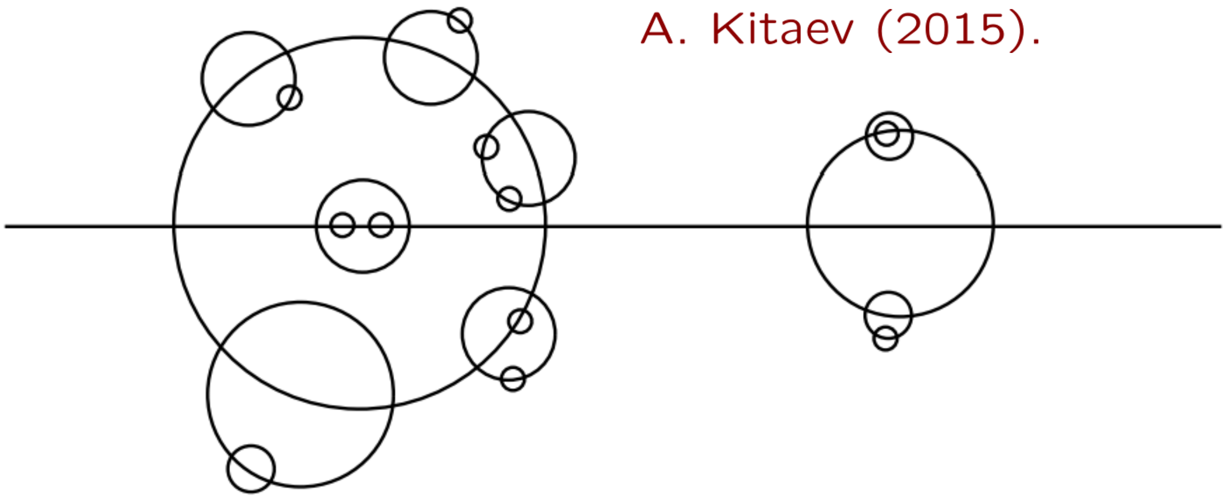
A general Hamiltonian would be $H = \frac{i}{2!} \sum_{i,j} \Gamma_{i,j} \chi_i \chi_j + \frac{1}{4!} \sum_{i,j,k,l} J_{i,j,k,l} \chi_i \chi_j \chi_k \chi_l$

$$\langle \Gamma_{ij}^2 \rangle = \frac{\Gamma^2}{N}$$

Feynman diagrams

Starting point: $\Gamma = 0$

Typical diagram for $G(\tau) = \langle \chi_i(\tau)\chi_i(0)\rangle$ at large N



A. Kitaev (2015).

Thanks to $\langle J_{ijkl}^2 \rangle = \frac{3! J^2}{N^3}$, the same in the thermodynamic limit!

Self-consistency equation for sum of the diagrams:

$$G(\omega) = \frac{1}{-i\omega - \Sigma(\omega)}, \quad \Sigma(\tau) = J^2 G(\tau)^3$$

Infrared limit

Sachdev, Ye (1993)

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathcal{S}}_i \cdot \hat{\mathcal{S}}_j \qquad \text{with } \hat{S} \text{ from SU(M)}$$

$$\hat{S} \to \text{Schwinger bosons}$$

In the infrared $Jt \gg 1$, take $-i\omega \to 0$:

$$G(\omega) = \frac{1}{-\Sigma(\omega)}, \quad \Sigma(\tau) = J^2 G(\tau)^3$$

Exact solution on the line ($T = 0$): $G(\tau) = \frac{\text{sign}\tau}{|\tau|^{1/2}}$

And on the circle: $G(\tau) = \frac{\text{sign}\tau}{\sin^{1/2}\left(\frac{\pi\tau}{\beta}\right)}$

Field theory

Parcollet, Georges, Sachdev (2001)

Kitaev (2015)

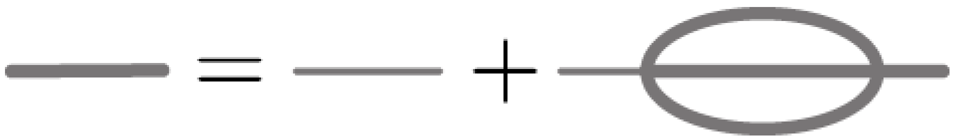
Average over disorder and integrate out Majoranas χ

$$Z = \int D[G,\Sigma] \exp(-S[G,\Sigma])$$

$$S(G,\Sigma) = -\frac{N}{2} \int_{-\beta/2}^{\beta/2} \int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{tr} \ln \left(\partial_\tau \delta^{ab} + \Sigma^{ab}_{\tau\tau'} \right) + \frac{J^2}{4} \left[G^{ab}_{\tau\tau'} \right]^4 + \Sigma^{ba}_{\tau'\tau} G^{ab}_{\tau\tau'} \right]$$

Saddle point equations

$$-(\partial_\tau + \Sigma) \cdot G = 1; \;\; \Sigma = J^2 [G]^3$$



with solutions (∂_τ neglected)

$$G^{ab}(\tau - \tau') = -\frac{b}{J^{1/2}} \frac{\delta^{ab} \mathrm{sign}(\tau - \tau')}{|\tau - \tau'|^{1/2}}$$

$$b = (4\pi)^{-1/4}$$

$$\Sigma^{ab}(\tau - \tau') = -b^3 J^{1/2} \frac{\delta^{ab} \mathrm{sign}(\tau - \tau')}{|\tau - \tau'|^{3/2}}$$

Symmetries of the action

$$S(G,\Sigma) = -\frac{N}{2}\int_{-\beta/2}^{\beta/2}\int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{tr}\ln\left(\partial_\tau \delta^{ab} + \Sigma^{ab}_{\tau\tau'}\right) + \frac{J^2}{4}\left[G^{ab}_{\tau\tau'}\right]^4 + \Sigma^{ba}_{\tau'\tau} G^{ab}_{\tau\tau'}\right]$$

$\partial_\tau \to 0$ action is invariant under reparametrization of time

$\tau \to f(\tau)$ with $f(\tau)$ monotonic differentiable

$$G(\tau,\tau') \to f'(\tau)^{1/4} G(f(\tau) - f(\tau')) f'(\tau')^{1/4}$$

$$\Sigma(\tau,\tau') \to f'(\tau)^{3/4} \Sigma(f(\tau) - f(\tau')) f'(\tau')^{3/4}$$

weakly broken by time derivative!

Symmetry of the mean field

$$G^{ab}(\tau - \tau') = -\frac{b}{J^{1/2}} \frac{\delta^{ab}\mathrm{sign}(\tau - \tau')}{|\tau - \tau'|^{1/2}}$$

$$\Sigma^{ab}(\tau - \tau') = -b^3 J^{1/2} \frac{\delta^{ab}\mathrm{sign}(\tau - \tau')}{|\tau - \tau'|^{3/2}}$$

Invariant under $f(\tau) = \frac{a\tau + b}{c\tau + d}$, $ad - bc = 1$

Infinite dimensional Goldstone mode manifold

Each $f \in \mathrm{Diff}/SL(2,R)$ generates a new solution:

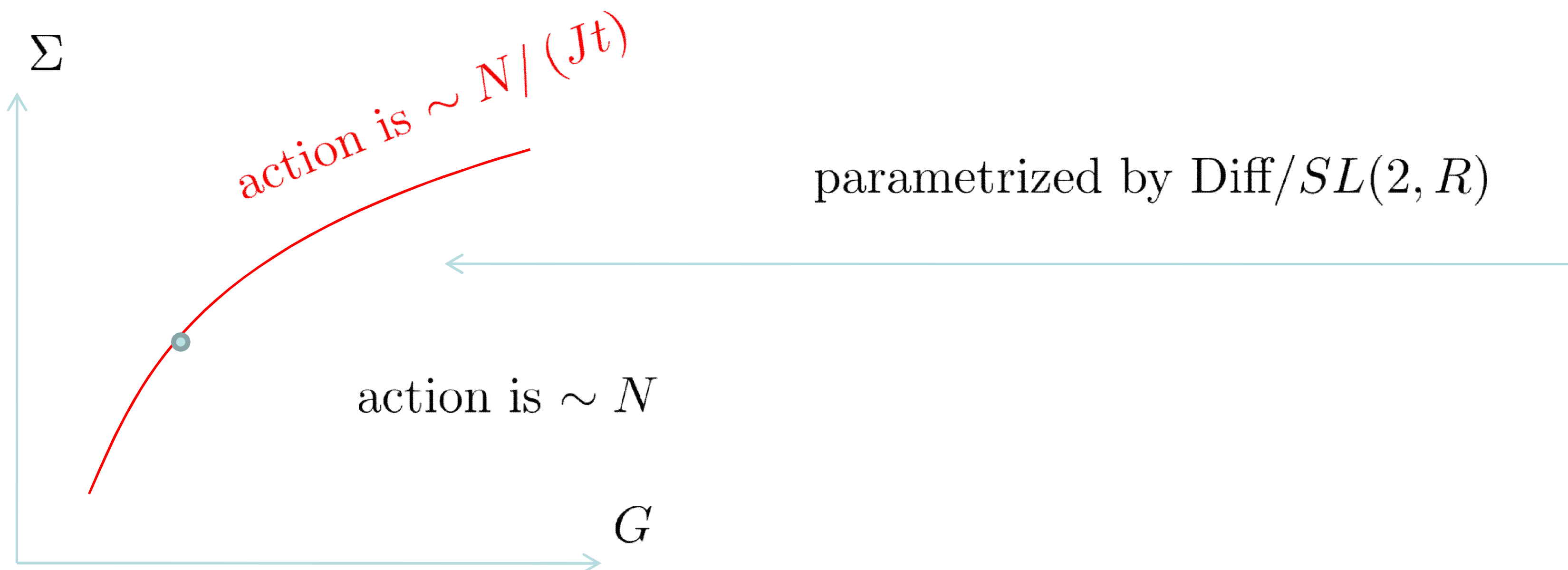
$$G_{\tau\tau'} = -\frac{b}{J^{1/2}} \mathrm{sign}(f(\tau) - f(\tau')) \frac{f'(\tau)^{1/4} f'(\tau')^{1/4}}{|f(\tau) - f(\tau')|^{1/2}}$$

$$\Sigma_{\tau\tau'} = -b^3 J^{1/2} \mathrm{sign}(f(\tau) - f(\tau')) \frac{f'(\tau)^{3/4} f'(\tau')^{3/4}}{|f(\tau) - f(\tau')|^{3/2}}$$

Large N action

beyond the strict IR the zero modes are slightly lifted

A. Kitaev (2015).
J. Maldacena, D. Stanford (2015).



SL(2, R) acts as $\tau \to \frac{a\tau+b}{c\tau+d}$

$$\mathrm{Sch}(\mathrm{f},\tau) \equiv \left(\frac{f''}{f'}\right)' - \frac{1}{2}\frac{f''^2}{f'^2}$$

Action for $\tau \to f(\tau)$: $S = -\#\frac{N}{J}\int d\tau \mathrm{Sch}(f,\tau)$

Reparametrization action

D. Bagrets, A. Altland, A. Kamenev (2016).

$$S(G,\Sigma) = -\frac{N}{2}\int_{-\beta/2}^{\beta/2}\int_{-\beta/2}^{\beta/2} d\tau d\tau' \left[\mathrm{tr}\ln\left(\partial_\tau \delta^{ab} + \Sigma^{ab}_{\tau\tau'}\right) + \frac{J^2}{4}\left[G^{ab}_{\tau\tau'}\right]^4 + \Sigma^{ba}_{\tau'\tau} G^{ab}_{\tau\tau'}\right]$$

Construction of effective action for low-cost reparametrizations

$$S(G,\Sigma) = \frac{N}{4}\mathrm{Tr}\left(\partial_\tau G \partial_\tau G\right) = -\frac{b^2 N}{16 J}\int d\tau d\tau' \frac{f'(\tau)^{3/2} f'(\tau')^{3/2}}{|f(\tau)-f(\tau')|^3}$$

Short-time cutoff generates the action for these reparametrization modes:

$$S[f] = \frac{M}{2}\int d\tau \left(\frac{f''(\tau)}{f'(\tau)}\right)^2 \qquad\qquad M = \frac{b^2}{32J} N \ln N$$

M of effective action

$$S[f] = \frac{M}{2} \int d\tau \left(\frac{f''(\tau)}{f'(\tau)} \right)^2$$

$$\rho(\epsilon) \propto \sinh\left(2\pi\sqrt{2M\epsilon}\right)$$

D. Bagrets, A. Altland, A. Kamenev (2017).

$$\rho(\epsilon) \propto \sinh\left(\frac{2\pi\sqrt{2}\sqrt{\epsilon/\epsilon_N}}{\ln 1/\eta_N}\right)$$

A. Garcia-Garcia, J. Verbaarschot (2017).

$$\bar{\epsilon}_N = \frac{JN}{16\sqrt{2}} + O(1) \quad \eta_N = 1 - \frac{32}{N} + O(\frac{1}{N^2})$$

$$M = \frac{m(N)}{32\sqrt{2}}\frac{N}{J}, \quad m(N) = 1 + O(\frac{1}{N})$$

SYK as Liuville Quantum Mechanics

$$G(f,\tau,\tau') = -\frac{b}{J^{1/2}} \left\langle \frac{f'(\tau)^{1/4} f'(\tau')^{1/4}}{|f(\tau)-f(\tau')|^{1/2}} \right\rangle_f$$

non-gaussian averaging

$$S[f] = \frac{M}{2} \int d\tau \left(\frac{f''(\tau)}{f'(\tau)} \right)^2 \qquad \Longrightarrow \qquad S_\phi = \frac{M}{2} \int (\phi')^2 \, d\tau$$

$$f'(\tau) = e^{\phi(\tau)}$$

$$G(\tau-\tau') = -\frac{b}{\sqrt{J}} \int \mathcal{D}\phi \frac{e^{\frac{1}{4}\phi(\tau)} e^{\frac{1}{4}\phi(\tau')}}{\sqrt{\int_{\tau'}^{\tau} d\tau'' e^{\phi(\tau'')}}} e^{-\frac{M}{2}\int d\tau'' \left[\phi'(\tau'')\right]^2}$$

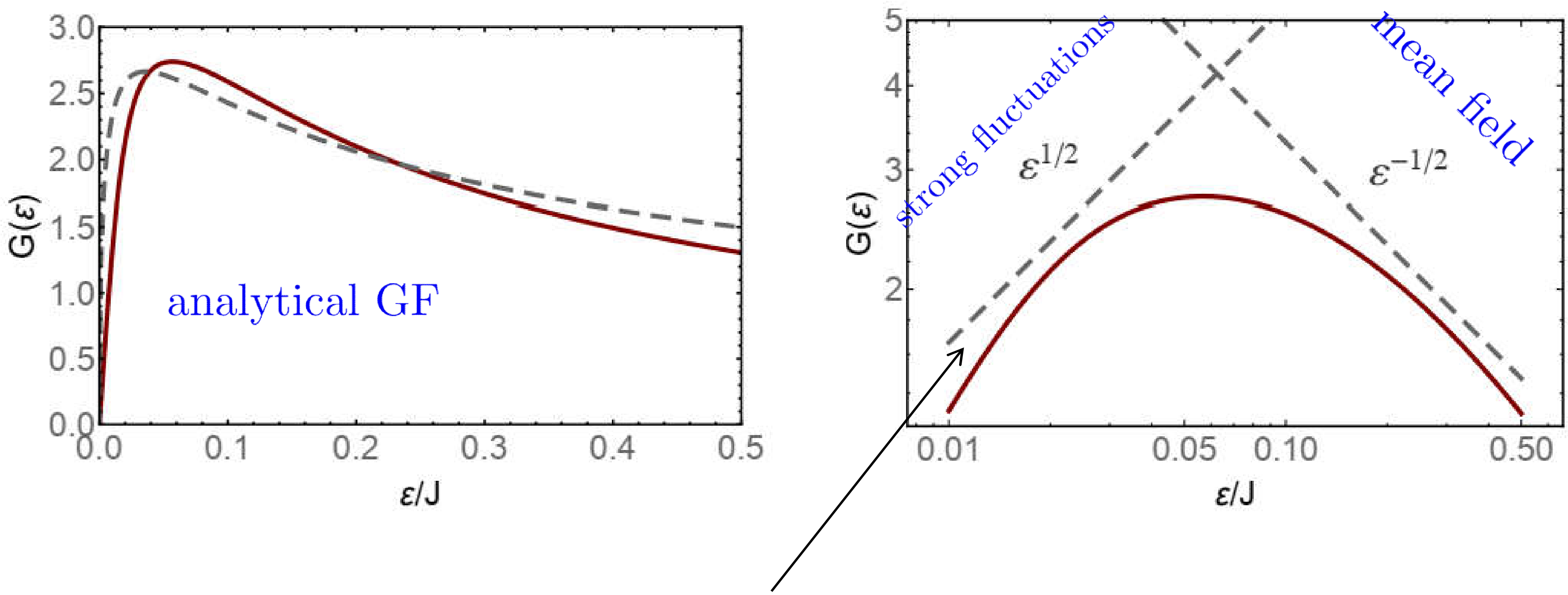
$$G(\tau-\tau') = -\frac{b}{\sqrt{\pi J}} \left\langle e^{\frac{1}{4}\left(\phi(\tau)+\phi(\tau')\right)} \int_0^\infty \frac{d\alpha}{\sqrt{\alpha}} e^{-\alpha \int_{\tau'}^{\tau} d\tau'' e^{\phi(\tau'')}} \right\rangle_\phi$$

QM with quench potential

Calculation of GF

D. Bagrets, A. Altland, A. Kamenev (2016).

$$G(\epsilon) = -\frac{ib}{\sqrt{J}}\left(\frac{2}{\pi M}\right)^{1/2}\int_0^\infty dk\,\frac{k\sinh(2\pi k)}{2\pi^2}\Gamma^2\left(\tfrac{1}{4}+ik\right)\Gamma^2\left(\tfrac{1}{4}-ik\right)\frac{2\epsilon}{E_k^2+\epsilon^2}$$



restoration of the full symmetry of the action at large times

Perturbed model

Motivation

- SYK model demonstrates non-FL behavior at all energies. In particular, there is no quasi-particle description of excited states.
- Extension to usual complex Fermions is straightforward at the saddle-point level
- Can one use SYK model as a basis to construct a theory of non-Fermi liquid state of interacting Fermions? Is SYK solution stable ($T = 0$) w.r.t. perturbations, quadratic in Fermions?

Quite a few recent papers argued that the answer is NO

(on the level of saddle-point approx.)

We study this problem beyond the saddle-point approximation

Previous results

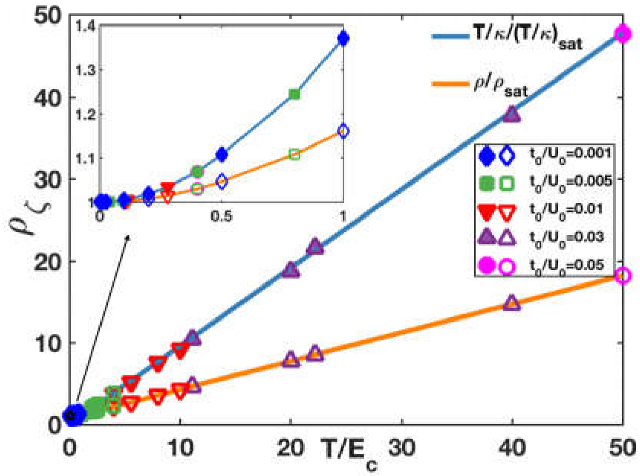
S.-K. Jian, H. Yao (2017)

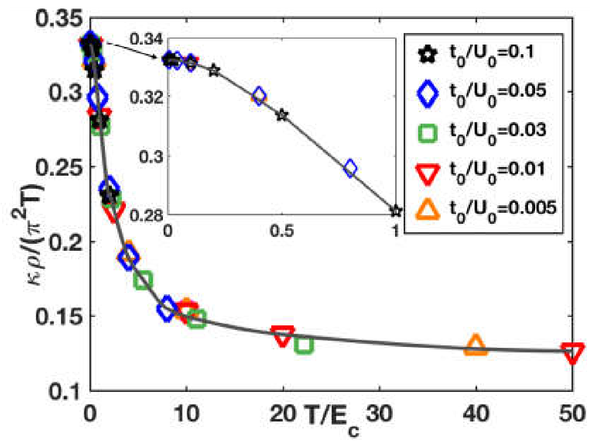
D. Chowdhury, Y. Werman, E. Berg, T. Senthil (2018)

X.-Y. Song, C.-M. Jian, L. Balents (2017)

$d-$dimensional array of SYK quantum dots

$$\mathcal{H} = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'} \qquad \overline{|U_{ijkl,x}|^2} = 2U_0^2/N^3 \text{ and } \overline{|t_{ij,x,x'}|^2} = t_0^2/N.$$





- Intermediate T regime exists, $T_* \ll T \ll J$ with non-FL behavior
- At lowest T FL is recovered. No soft mode fluctuations are taken into account!

Perturbed model

$$H = \frac{1}{4!}\sum_{i,j,k,l} J_{i,j,k,l}\chi_i\chi_j\chi_k\chi_l + \frac{i}{2!}\sum \Gamma_{i,j}\chi_i\chi_j$$

$$\langle J_{ijkl}^2\rangle = \frac{3!J^2}{N^3},\ \langle \Gamma_{ij}^2\rangle = \frac{\Gamma^2}{N}$$

?

Pure SYK_4 | Pure SYK_2

$\gamma = \Gamma/J$

0 | ∞

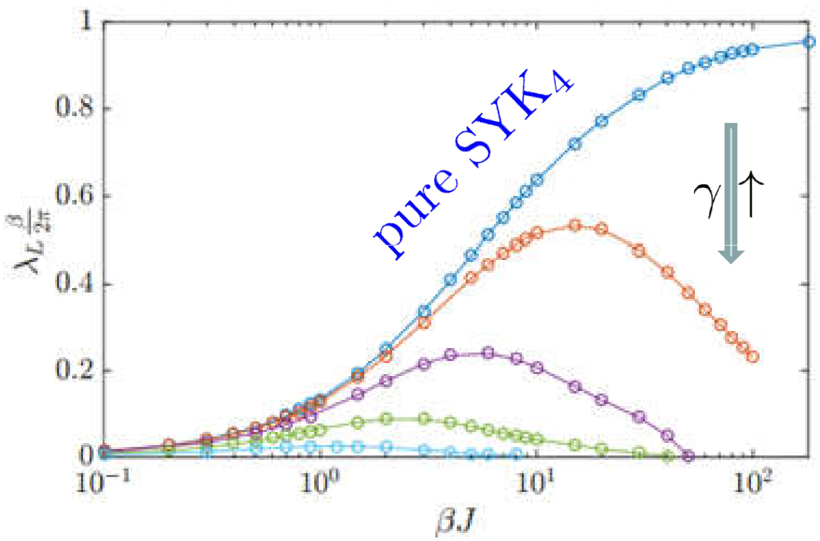
Wigner-Dyson level statistics	poisson level statistics
ergodicity in the Fock space	localization in the Fock space
non-FL behavior of GF at all scales	FL behavior of GF at small ϵ

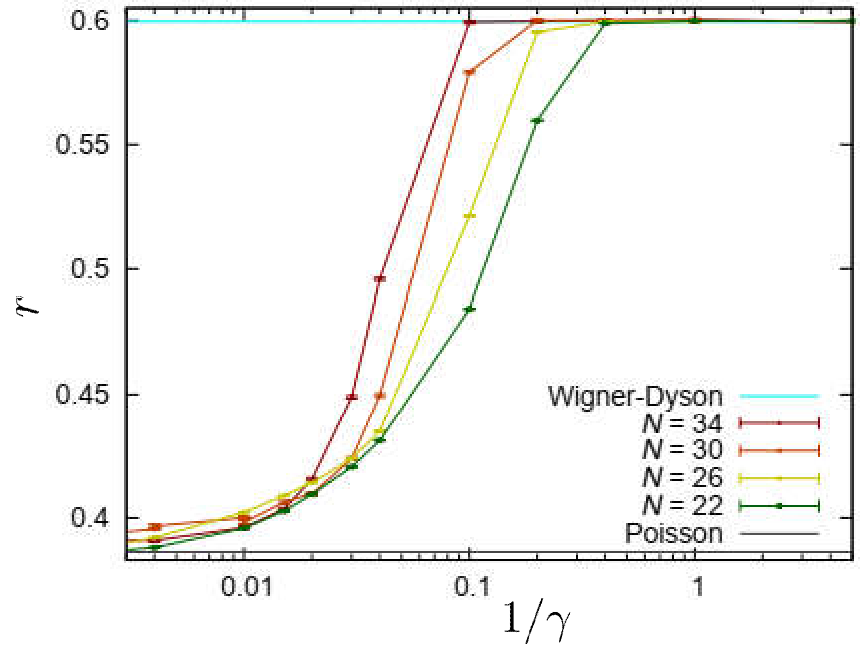
Previous numerics

A. Garcia-Garcia, B. Loureiro, A. R.-Bermudez, M. Tezuka (2017)

saddle-point $N \to \infty$

exact diagonalization, middle of the band





$\gamma_c(T = 0) = 0$

$\gamma_c(T = \infty) \approx 67$

Chaotic phase is unstable at $T = 0$ wrt quadratic term in the $N \to \infty$ limit

Let us study the Green function $G(\tau)$

Simple estimates-1

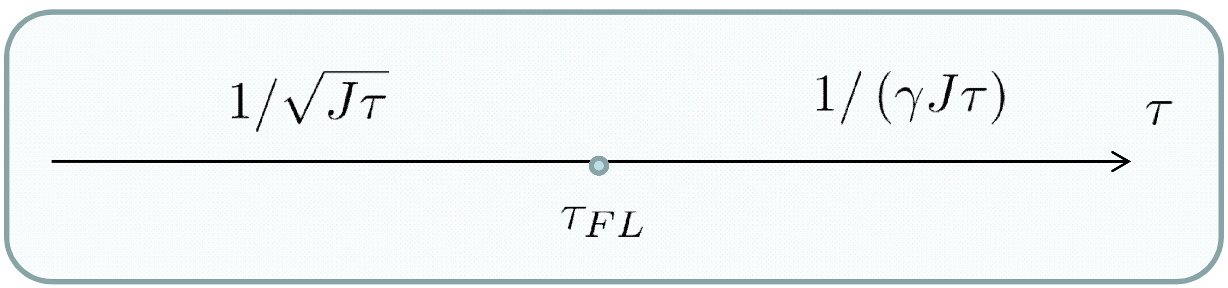
Scaling analysis at the saddle-point level

$$\partial_\tau G_{\tau\tau'} - \int d\tau'' \Sigma_{\tau\tau''} G_{\tau''\tau'} = \delta(\tau - \tau') \qquad \Sigma_{\tau\tau'} = J^2 G^3_{\tau\tau'} + \Gamma^2 G_{\tau\tau'}$$

$$G_0(\tau) \propto \frac{1}{\sqrt{J\tau}} \qquad \frac{\Gamma^2 G_0(\tau)}{J^2 G_0^3(\tau)} \propto \left(\frac{\Gamma}{J}\right)^2 J\tau$$

Grows with τ: instability in the infra-red

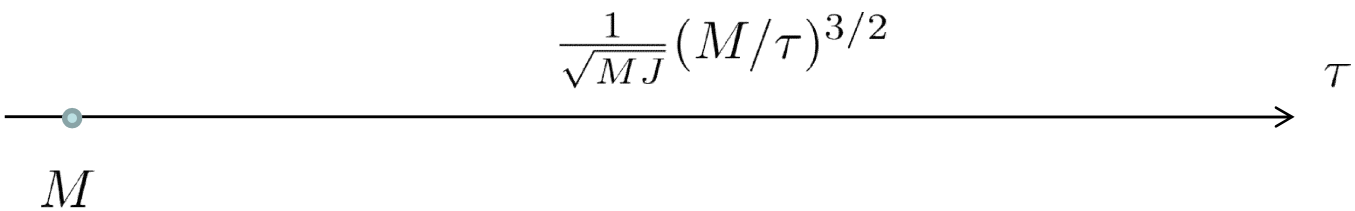
$$\tau_{FL} = 1/\left(J\gamma^2\right), \quad \gamma = \Gamma/J$$



At finite N there is $\tau \propto M$ scale far to the right $\longrightarrow$

Simple estimates-2

Scaling analysis in the infrared limit



D. Bagrets, A. Altland, A. Kamenev (2016). $\langle G^p(\tau)\rangle \propto \frac{M^{(3-p)/2}}{J^{p/2}}\frac{1}{\tau^{3/2}}$

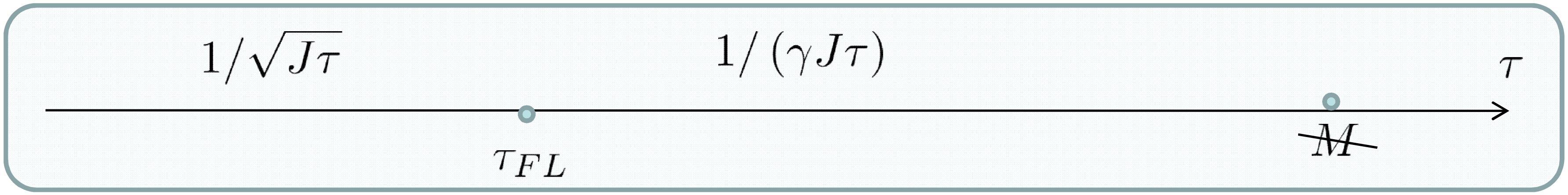
$$\frac{\Gamma^2 G_0(\tau)}{J^2 G_0^3(\tau)} \propto \left(\frac{\Gamma}{J}\right)^2 JM \sim \gamma^2 N$$

Does not grow with τ: posible stability in the infrared

Naively: effect of quadratic term is small as long as $\gamma < 1/\sqrt{N}$

Perturbation theory for GF

$$\gamma \gtrsim 1/\sqrt{N}$$



What about smaller γ?

$$G_{\tau\tau'}\left[\phi(\tau)\right] = \frac{1}{\sqrt{2J\sqrt{\pi}}} \mathrm{sign}(\tau-\tau') \frac{e^{\phi(\tau)/4} e^{\phi(\tau')/4}}{\left|\int_{\tau'}^{\tau} e^{\phi}(\tilde{\tau}) d\tilde{\tau}\right|^{1/2}}$$

$$S_\phi = \frac{M}{2}\int (\phi')^2 d\tau. \qquad S_2 = \int d\tau d\tau' \frac{\Gamma^2}{2} G^2_{\tau\tau'}$$

Perturtbative expansion in $\gamma = \Gamma/J$

$$\delta G(\tau) = -\langle G_{\tau,0}[\phi] S_2[\phi]\rangle_0 + \langle G_{\tau,0}\rangle_0 \langle S_2\rangle_0$$

$\int d\tau_1, \int d\tau_2$

Technicalities

$$\delta G(\tau) = -\langle G_{\tau,0}[\phi] S_2[\phi]\rangle_0 + \langle G_{\tau,0}\rangle_0 \langle S_2\rangle_0, \quad S_2 = \int d\tau d\tau' \frac{\Gamma^2}{2} G^2_{\tau\tau'}$$

No Wick theorem, need for explicit calculation of different time orderings

1	2	3	4	5	6
$\tau_2, \tau_1, 0, \tau$	$\tau_2, 0, \tau_1, \tau$	$\tau_2, 0, \tau, \tau_1$	$0, \tau_2, \tau_1, \tau$	$0, \tau_2, \tau, \tau_1$	$0, \tau, \tau_2, \tau_1$

Need for $\langle G^n(t_1, t_2) G^m(t_3, t_4)\rangle$

$$\int_0^\infty \alpha^{2n-1} d\alpha \int_0^\infty \beta^{2m-1} d\beta \left\langle e^{\frac{1}{4}(n\phi_1 + n\phi_2 + m\phi_3 + m\phi_4)} e^{-\alpha \int_{t_2}^{t_1} e^{\phi(\tilde{t})} d\tilde{t} - \beta \int_{t_4}^{t_3} e^{\phi(\tilde{t})} d\tilde{t}} \right\rangle_\phi$$

$$\phi_i = \phi(t_i)$$

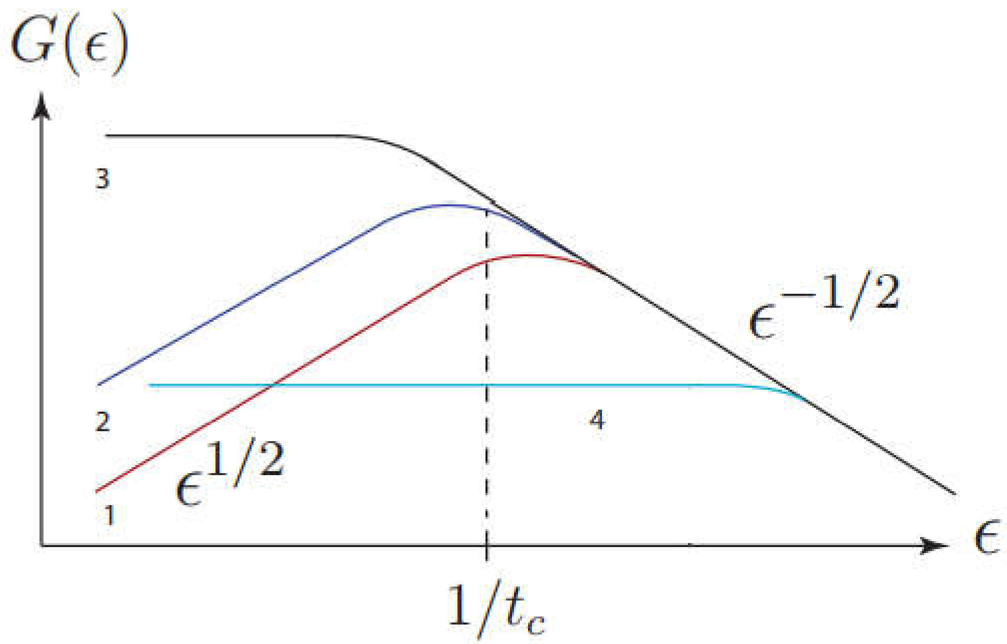
contributions 1, 6 cancel completely

Same QMech problem but with several quenches

Result for GF

$$\delta G(\tau) = cN\sqrt{MJ}\gamma^2(\tau/M)^{-\frac{3}{2}}$$

$\delta G/G \approx 0.081 N^2\gamma^2$ Relevant parameter in the infra-red limit: $N\Gamma/J$

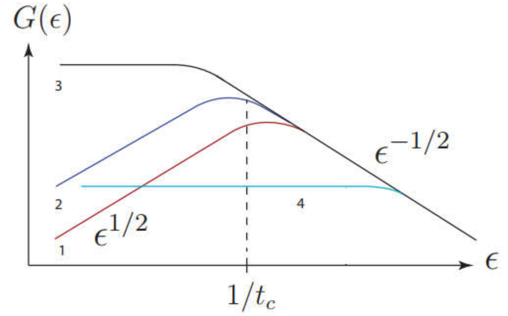


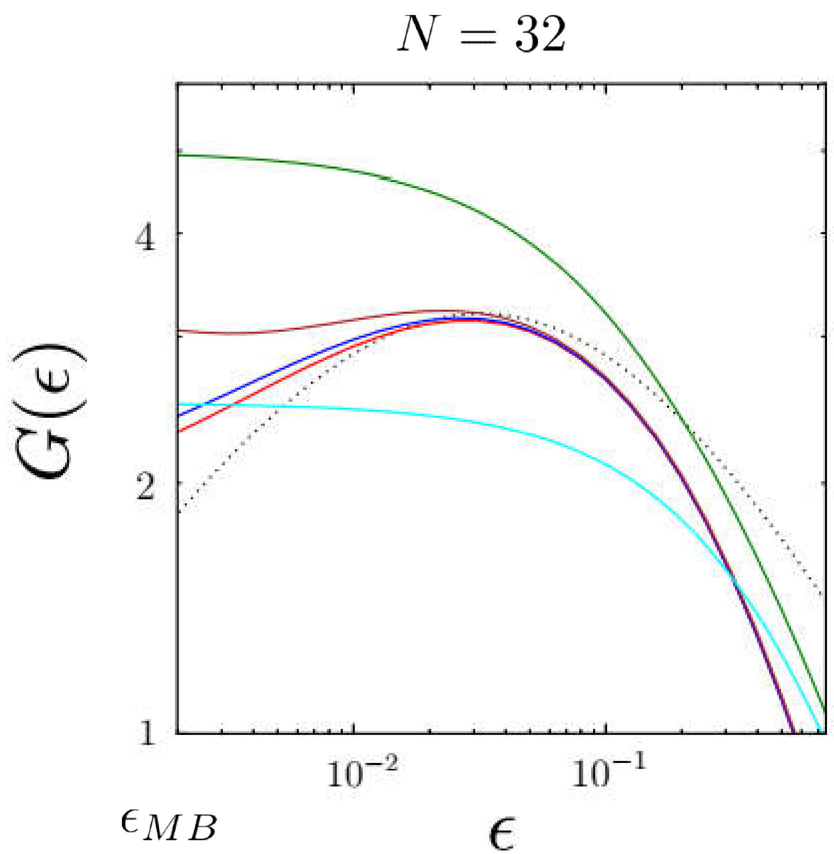
$SYK_4 + \gamma \cdot SYK_2$ at $\gamma \uparrow$

1. $\gamma = 0$ } nFL
2. $\gamma \ll 1/N$ } nFL
3. $\gamma \sim 1/N$ } FL
4. $\gamma \gtrsim 1/\sqrt{N}$ } FL

$\longleftarrow$ Many-body level spacing is infenitesimal $2^{-N/2} \rightarrow 0$

Numerics





dotted. Bagrets, Altland, Kamenev (2016)

$\gamma = 0$, exact diagonalization

$\gamma = 0.01$, exact diagonalization

$\gamma = 0.03$, exact diagonalization

$\gamma = 0.2$, mean field

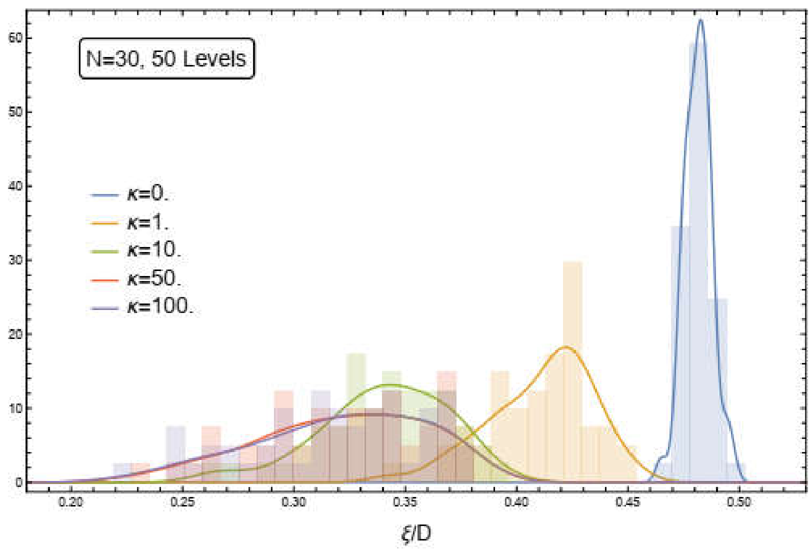
$\gamma = 0.4$, mean field

More recent numerics

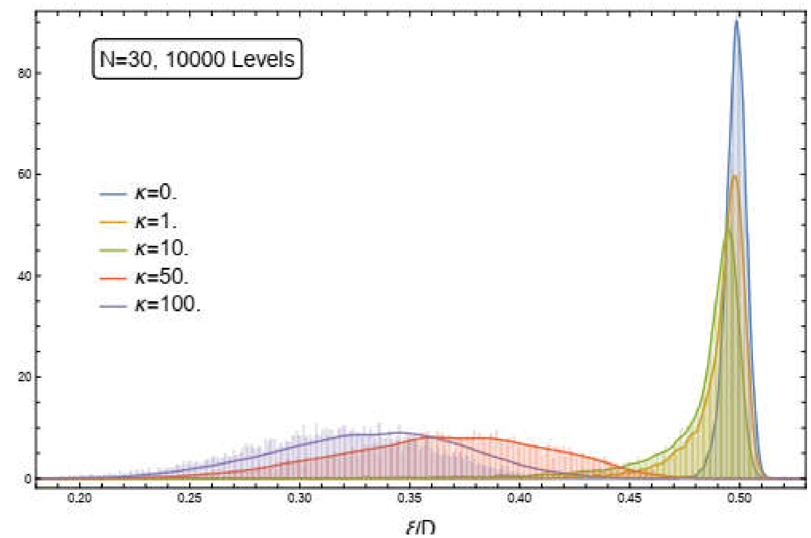
Chaotic-integrable transition at various energy densities

T. Nosaka, D. Rosa, J. Yoon (2018)

- Temperature-dependence of the transition: $\kappa_c \sim 1$ for low-lying states and at $\kappa_c \sim 15$ for highly excited states
- Localization in the Fock space as probed by many-body wavefunctions and spectral statistics



Low-lying states



Highly excited states

More recent numerics

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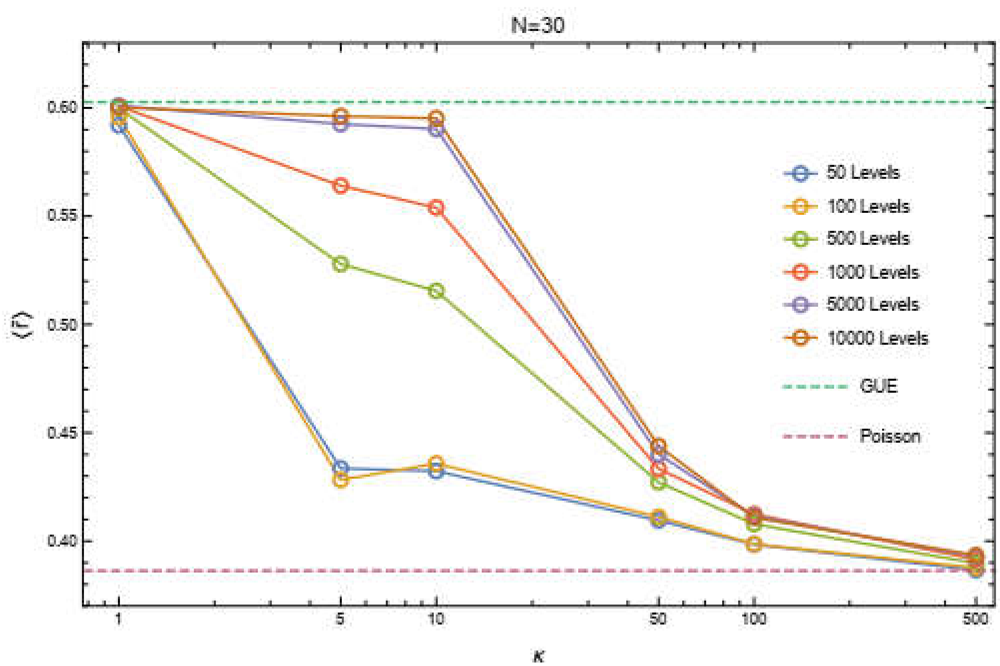

Conclusions and perspective

- Judging from the GF, Non-FL ground state ($T = 0$) is stable in nonzero area of the parameter space of $\mathrm{SYK}_4 + \mathrm{SYK}_2$. This area decreases as number of Fermions N increases

- Consistent with numerically observed transition in the spectral and many-body wavefunctions at lowest energy densities

- Extension of the analysis for system of complex Fermions and for spatially extended system is of primary interest

- Treatment beyond perturbation theory is desirable: can effective action for SYK_2 terms be derived?

- Can SYK-like model be realized in materials with Cooper interaction, tending to create non-trivial pairing state and strong disorder, suppressing the pairing?